LECTURE 3

EARTHQUAKES: DETECTION,

LOCATION & FOCAL GEOMETRY

EARTHQUAKE LOCATION

Retrieved *(Inverted)* from **arrival times** of BODY (principally *P*) waves

The problem consists of determining

4 parameters

Epicenter { * Latitude, * Longitude }

* Latitude
* Longitude
[* Depth]

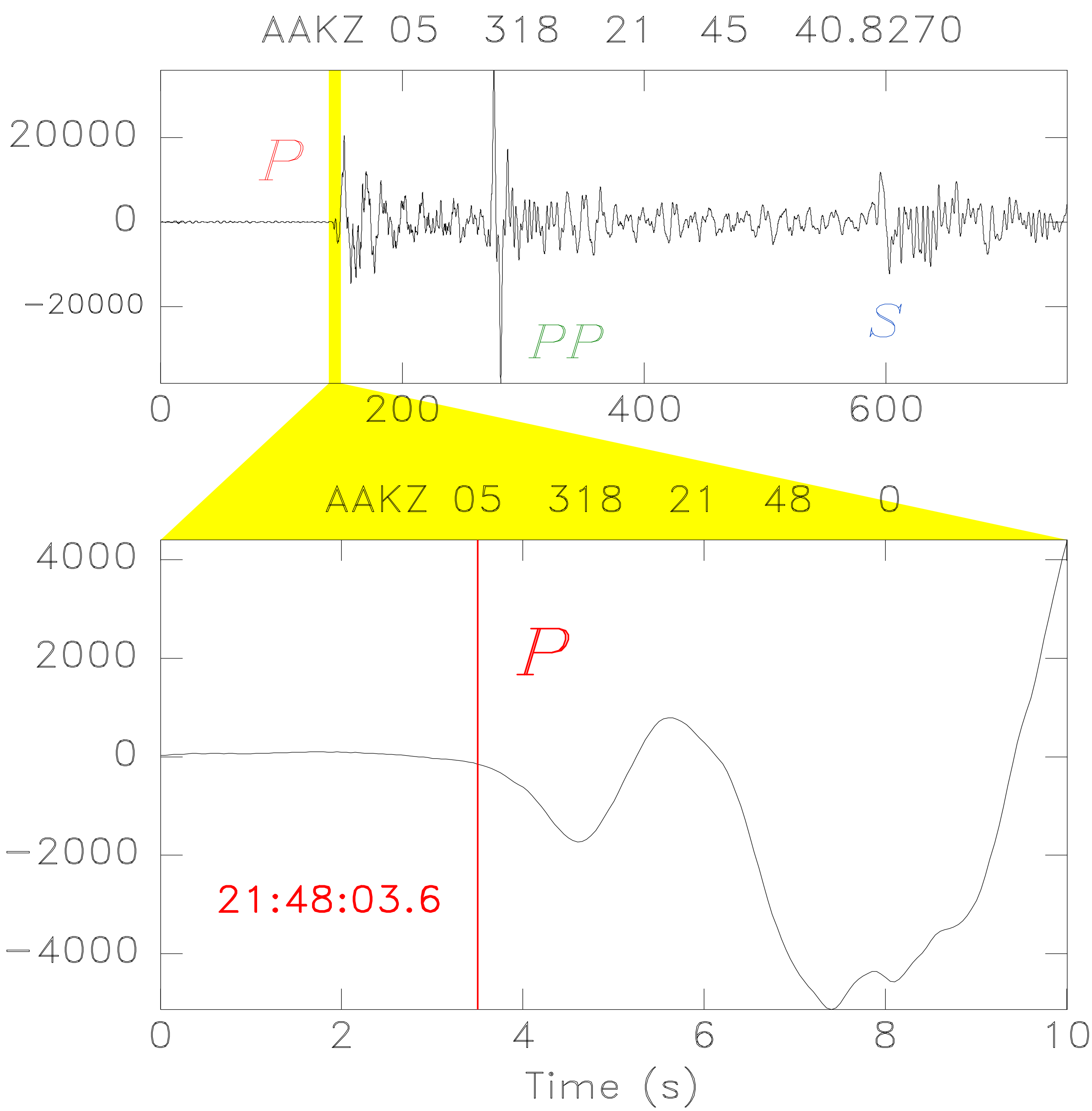
} **Hypocenter** (Latitude, Longitude, Depth)

* Origin Time

P times are usually easiest to pick

EXAMPLE: **JAPAN SEA Event, 14 NOV 2005**

Station AAK (Ala Archa, Kyrgyzstan); $\Delta = 52.4°$



→ **Gather such data for many stations**

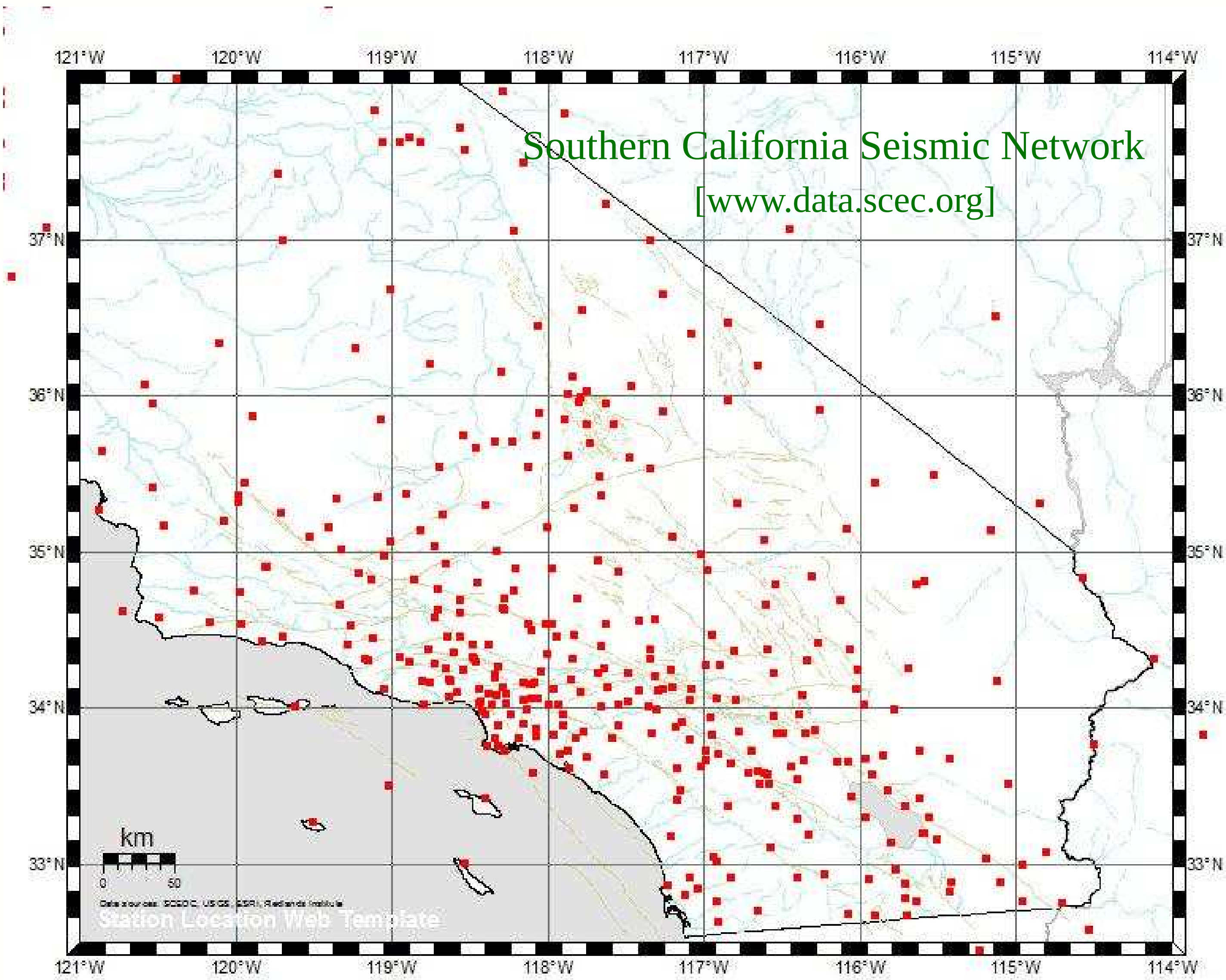
Obtain dataset of *observed* arrivals $\{ o_i \}$.

IN THE NEAR FIELD

- In the presence of a dense array, the

  Easiest, Simplest, Crudest Algorithm

  consists of identifying



Epicenter ≈ Station with Earliest Arrival

IN THE FAR FIELD

- *Position of problem:*

  Retrieve

  Latitude λ
  Longitude ϕ
  Depth h
  Origin time t_0

  from dataset $\{ o_i \}$.

- P –wave arrival times can be computed as functions of source and station parameters (Latitude and Longitude Λ_i , Φ_i) using a *model of Earth structure*.

$$c_i = f\ (\lambda, \phi, h, t_0; \Lambda_i , \Phi_i)$$

- *DIFFICULTY*:

  Function f is ***NON – LINEAR***.

LINEARIZING the PROBLEM

- Assume *Trial Solution*

$$\lambda^0 , \phi^0 , h^0 , t_0^0$$

and compute a set of *predicted arrival times* $\{c_i\}$ for that solution, based on a chosen Earth model (*Jeffreys-Bullen*, *PREM*, *iaspei91*, etc.).

→ If all the data were perfect (no noise), as well as the model, and we had guessed the right solution, then for all i, we should have $c_i = o_i$.

- Define *RESIDUALS*

$$\delta t_i \ = \ o_i - c_i \ = \ (o - c)_i$$

Hopefully, the δt_i are small compared with the propagation times $c_i - t_0^0$.

LINEARIZING the PROBLEM (2)

- Then try improving the solution from $\{\lambda^0, \phi^0, h^0, t_0^0\}$ to $\{\lambda^1, \phi^1, h^1, t_0^1\}$

$$\begin{pmatrix} \lambda^1 \\ \phi^1 \\ h^1 \\ t_0^1 \end{pmatrix} = \begin{pmatrix} \lambda^0 \\ \phi^0 \\ h^0 \\ t_0^0 \end{pmatrix} + \begin{pmatrix} \delta\lambda \\ \delta\phi \\ \delta h \\ \delta t_0 \end{pmatrix}$$

Again, we expect the terms $\delta \cdots$ to be small, so that the change in each c_i is simply

$$\delta c_i = \frac{\partial f}{\partial \lambda} \cdot \delta\lambda + \frac{\partial f}{\partial \phi} \cdot \delta\phi + \frac{\partial f}{\partial h} \cdot \delta h + \frac{\partial f}{\partial t_0} \cdot \delta t_0$$

- If we know the function f (*"direct problem"*), we should be able to compute the partial derivatives such as $\frac{\partial f}{\partial \lambda}$.

LINEARIZING the PROBLEM (3)

- Ideally, we would like, for each station, δc_i to be exactly $\delta t_i = (o - c)_i$, so we seek to solve

$$\begin{bmatrix} \cdots \\ \cdots \\ \cdots \\ \cdots \\ \delta c_i \\ \cdots \\ \cdots \\ \cdots \\ \cdots \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \\ & A & \\ \\ \\ \\ \\ \end{bmatrix} \cdot \begin{bmatrix} \delta \lambda \\ \delta \phi \\ \delta h \\ \delta t_0 \end{bmatrix} = \begin{bmatrix} \cdots \\ \cdots \\ \cdots \\ \cdots \\ \delta t_i \\ \cdots \\ \cdots \\ \cdots \\ \cdots \end{bmatrix}$$

LINEARIZING the PROBLEM (4)

The problem has been *LINEARIZED* but it is still *OVER-DETERMINED* as ***A*** is a very tall matrix (4 columns (4 unknowns) and tens or hundreds of rows (data points)).

→ It can be solved by the *classical LEAST−SQUARES algorithm:*

$$\begin{bmatrix} \delta\lambda \\ \delta\phi \\ \delta h \\ \delta t_0 \end{bmatrix} = (\mathbf{A}^{\mathbf{T}}\mathbf{A})^{-1} \cdot \mathbf{A}^{\mathbf{T}} \begin{bmatrix} \cdots \\ \cdots \\ \cdots \\ \cdots \\ \delta t_i \\ \cdots \\ \cdots \\ \cdots \\ \cdots \end{bmatrix}$$

- Note that $(\mathbf{A}^{\mathbf{T}}\mathbf{A})$ is a 4×4 matrix, and $\mathbf{A}^{T} \cdot \delta t$ is a 4−dimensional vector.

DETAILED LOOK at the MATRIX A

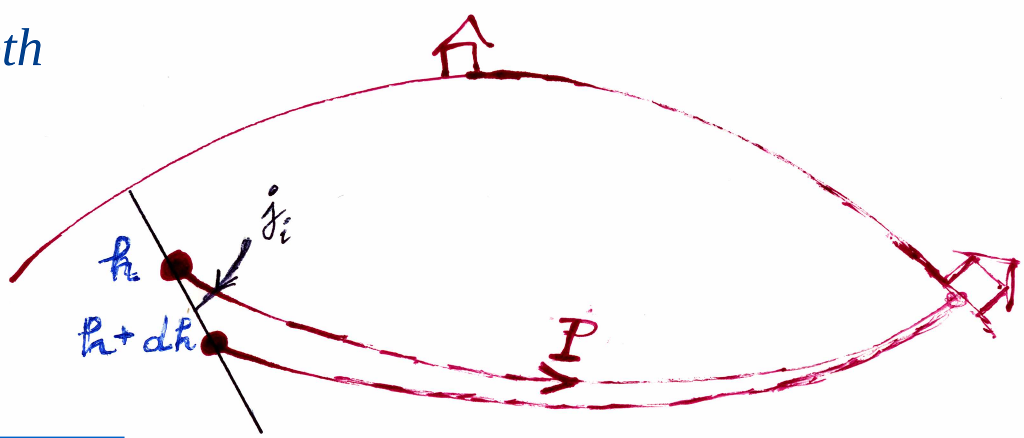
The elements of **A** are the partial derivatives of the arrival times c_i at station i with respect to a change in a source parameter

- *An easy case*

$$\frac{\partial c_i}{\partial t_0} = 1 \qquad \text{for all } i$$

Otherwise, for a spherical Earth, the travel-time t_P is function of the angular distance Δ to the station, and of the source depth, h.

- *Source depth*



$$\frac{\partial c_i}{\partial h} = -\frac{\cos j_i}{V^P(h)}$$

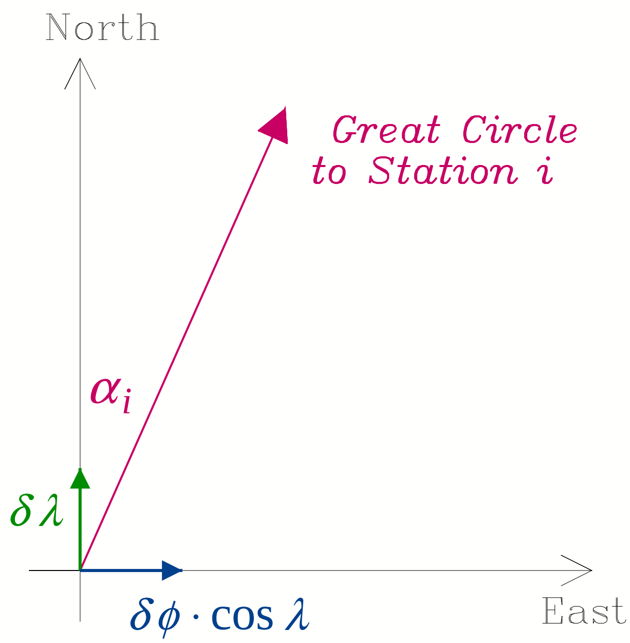
j is itself a function of Δ and h

NOTE that, at teleseismic distances, j is always a small angle...

DETAILED LOOK at the MATRIX A (2)

- *Latitude and Longitude*

  If we change the latitude by $\delta\lambda$, we move the epicenter North by an amount $\delta\lambda \cdot l_{deg.}$ where $l_{deg.} = 111.195$ km is the length of one degree at the Earth's surface.



  Thus, we change the distance to the station i by $\delta\Delta_i = -\,\delta\lambda \cdot \cos\alpha_i$, and

$$\frac{\partial c_i}{\partial \lambda} = -\cos\alpha_i \cdot \frac{\partial T_P}{\partial \Delta}$$

  If we change the longitude by $\delta\phi$, we move Eastwards, but only by $\delta\phi \cdot \cos\lambda \cdot l_{deg.}$, so that

$$\frac{\partial c_i}{\partial \phi} = -\sin\alpha_i \cdot \cos\lambda \cdot \frac{\partial T_P}{\partial \Delta}$$

IN SUMMARY

- We can compute all the partial derivatives
- We can compute the matrix **A**
- We can compute $(\mathbf{A}^T \mathbf{A})$ and invert it
- We can find the *"best"* change in earthquake source parameters to minimize the new residuals
- We can iterate the process until the solution stabilizes

LIMITATIONS of THIS ALGORITHM

- The matrix can be inverted only if it is

  NON – SINGULAR

  [in practice *NOT APPROACHING SINGULARITY*]

- The matrix is singular if 2 rows (or columns) are identical.

- Recall

$$\frac{\partial c_i}{\partial t_0} = 1 \quad \text{and} \quad \frac{\partial c_i}{\partial h} = -\frac{\cos j_i}{V^P(h)}$$

  If all the stations are at the same distance, then all j_i are the same and the two partials are proportional.

  SINGULARITY !

- In practice, if all stations are *far away*, then all j_i are small ($< 10°$; rays all take off nearly vertically at the source), all $\cos j_i \approx 1$, and one has

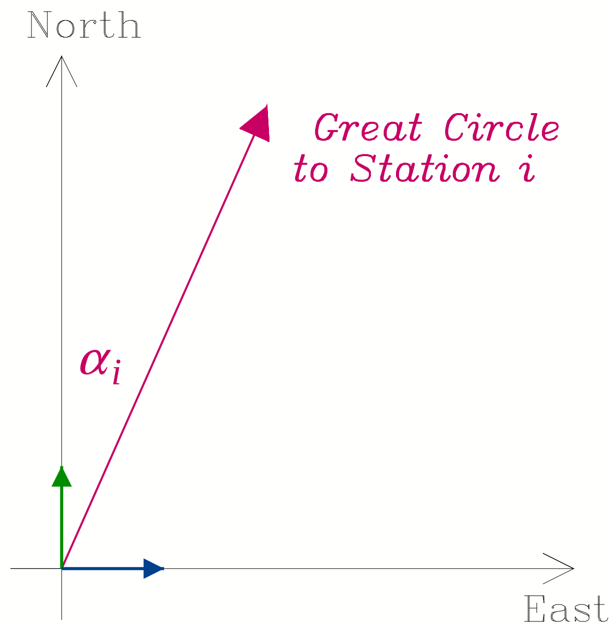
  PERFECT TRADE-OFF BETWEEN O.T. and DEPTH

  In general, the inversion becomes unstable. The only way out is to

  CONSTRAIN the DEPTH...

SIMILARLY

- Recall



$$\frac{\partial c_i}{\partial \lambda} = -\cos\alpha_i \cdot \frac{\partial T_P}{\partial \Delta}$$

and

$$\frac{\partial c_i}{\partial \phi} = -\sin\alpha_i \cdot \cos\lambda \cdot \frac{\partial T_P}{\partial \Delta}$$

If all stations are in [approximately] the same azimuth. the two columns of partials are proportional, and the matrix features [or approaches] **singularity.**

→ *STABLE LOCATIONS REQUIRE*

A GOOD AZIMUTHAL COVERAGE

[This is usually not an issue for large events]

INFLUENCE of TRIAL SOLUTION

- If dataset is global, any trial solution (even the antipodes of the true epicenter) will lead to a converging algorithm [*Okal and Reymond*, 2003].

- In practice, one can always use the station with earliest arrival as a trial epicenter.

ONE−STATION ALGORITHMS

Detection and Location

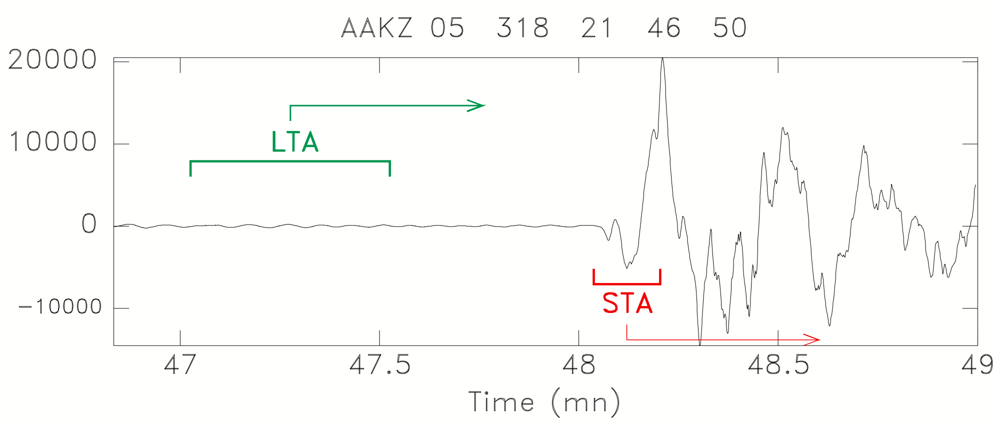
→ Enhance performance of single station/observatory

- *Detection Algorithms*

Generally based on the monitoring of energy in the ground (or velocity) motion at the station.

* Define a *SHORT-TERM AVERAGE* over a short sliding window

* Compare it with a *delayed LONG-TERM AVERAGE.*



- When $\frac{E \text{ in } STA}{E \text{ in } LTA}$ exceeds a given threshold,

TRIGGER DETECTION

* Earthquake spectrum is generally *WHITE*, so do this in *SEVERAL FREQUENCY BANDS.*

Coherence across spectrum is required to trigger detection

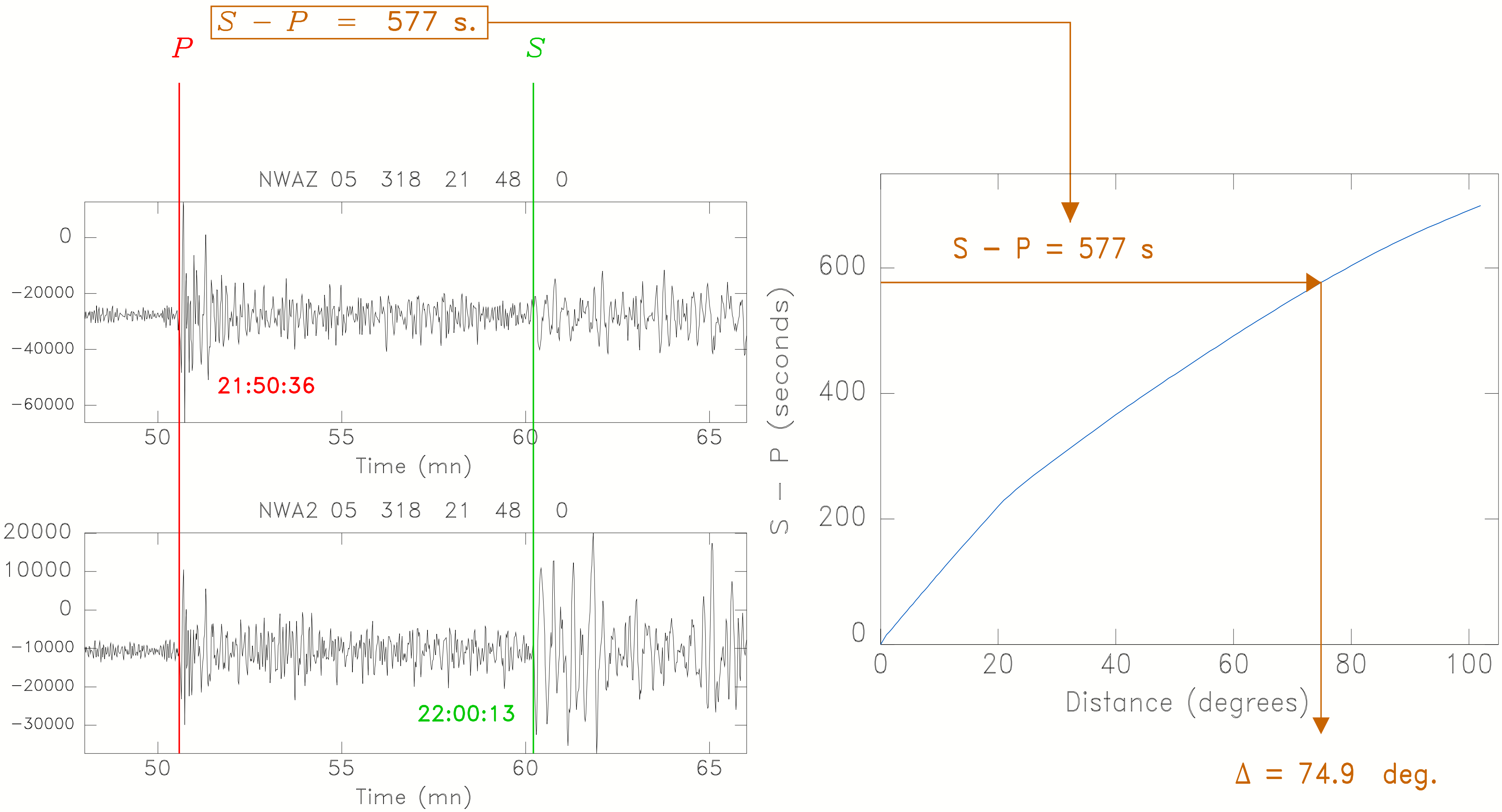
[or across several stations of a local network, when available to prevent triggering on human noise.]

→ Arrival times o_i can be defined by evolution of E_{STA}/E_{LTA}.

SINGLE-STATION LONG-PERIOD LOCATION

- *IDEA ONE*

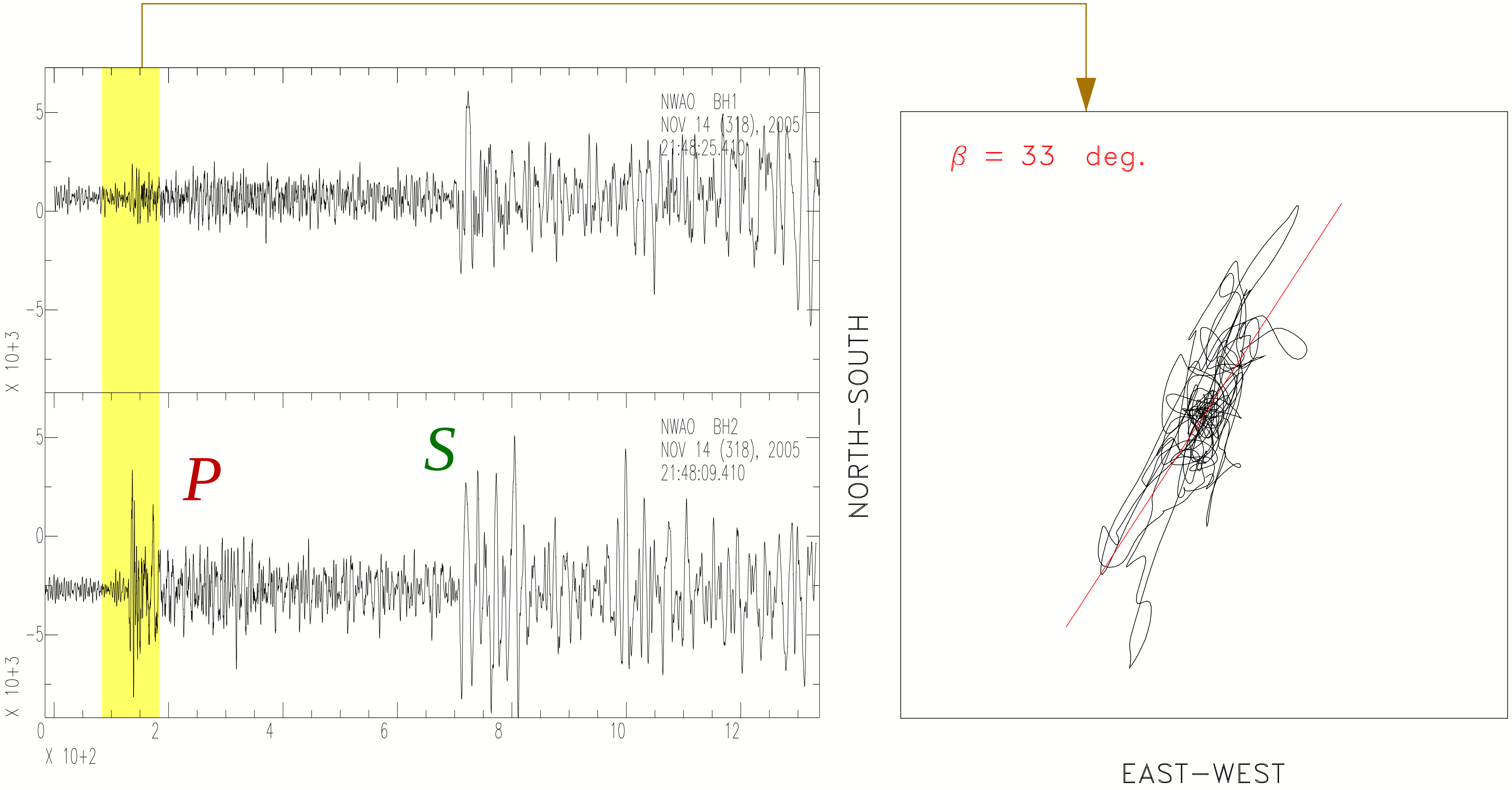
" *S* – *P* " interval between *P* and *S* waves can give *DISTANCE*



SINGLE-STATION LONG-PERIOD LOCATION

- *IDEA TWO*

Polarization of *P* wave can give *AZIMUTH of ARRIVAL*, *β*

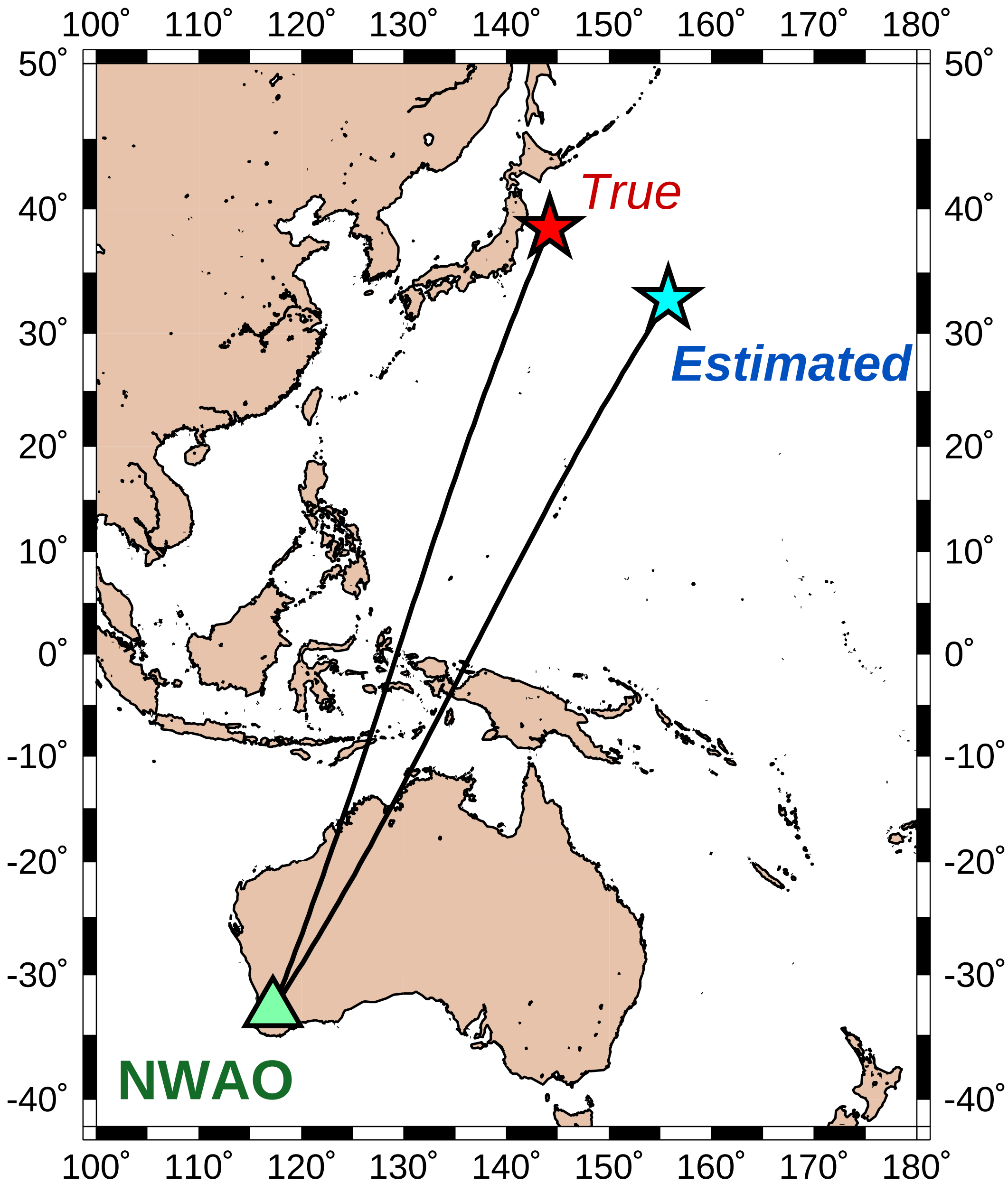


SINGLE-STATION LONG-PERIOD LOCATION

- Combine *DISTANCE and BACK AZIMUTH* to obtain

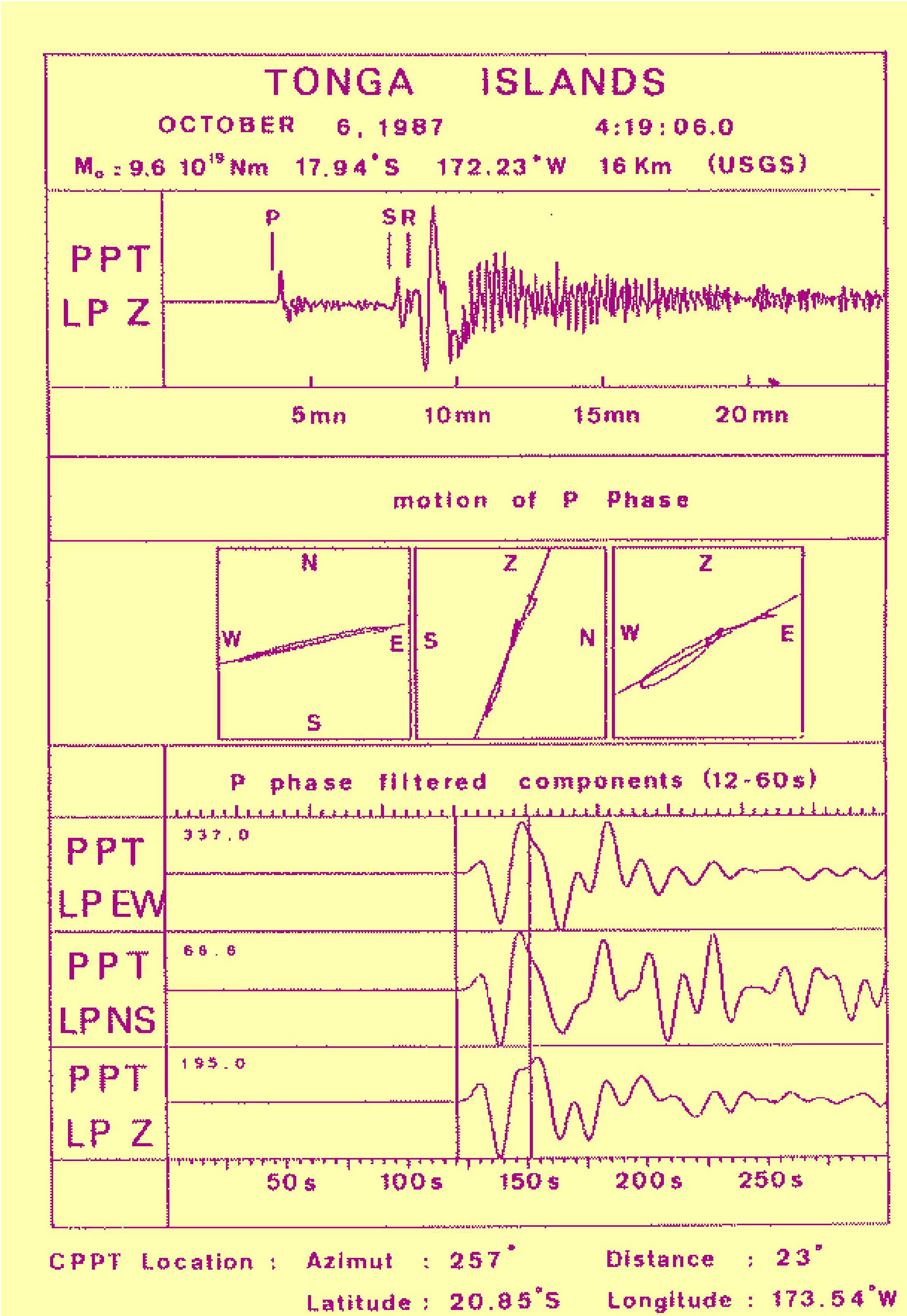
Estimate of Epicenter

14 NOV 2005



EXAMPLE of SINGLE-STATION LONG-PERIOD LOCATION

TREMORS — *Reymond et al.* [1991]



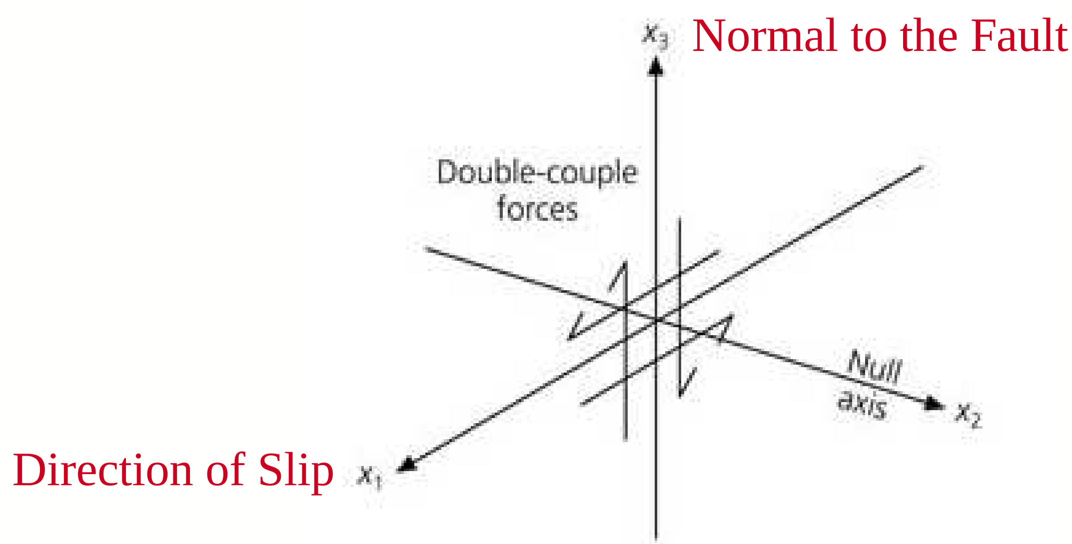
Earthquake located about 300 km from true epicenter

EARTHQUAKE SOURCE GEOMETRY

From Single Force to Double-Couple

The physical representation of an earthquake source is a system of forces known as a *Double-Couple*, the direction of the forces in each couple being the direction of slip on the fault and the direction of the normal to the fault plane.

Single Force



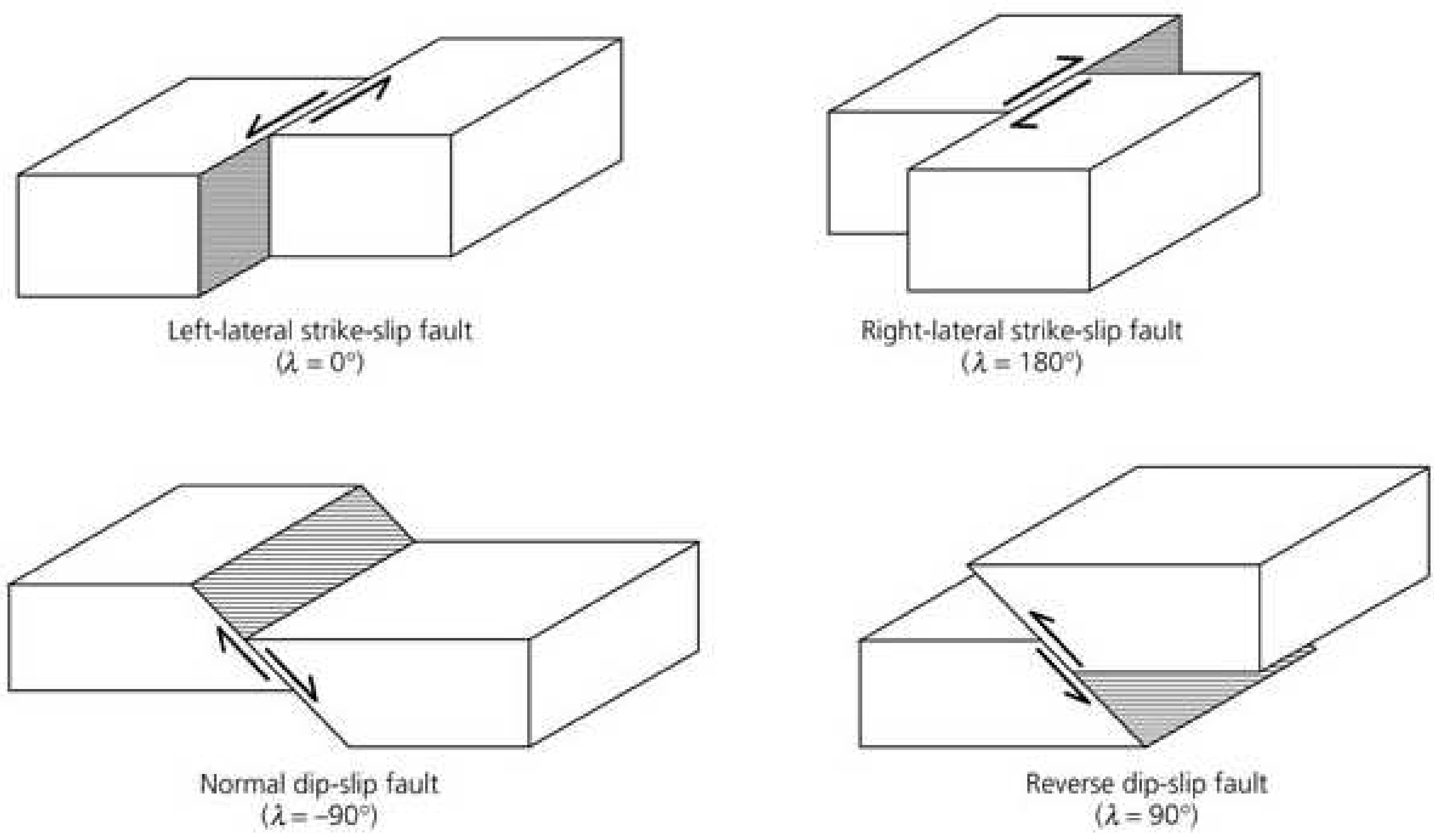
Double Couple

[*Stein and Wysession*, 2002]

Mathematically, the system of forces is described by a *Second-Order Symmetric Deviatoric TENSOR* (3 angles and a scalar).

EARTHQUAKE SOURCE GEOMETRY

The focal geometry of earthquakes can vary depending on the orientation of the double-couple representing the source. Here are some basic examples:



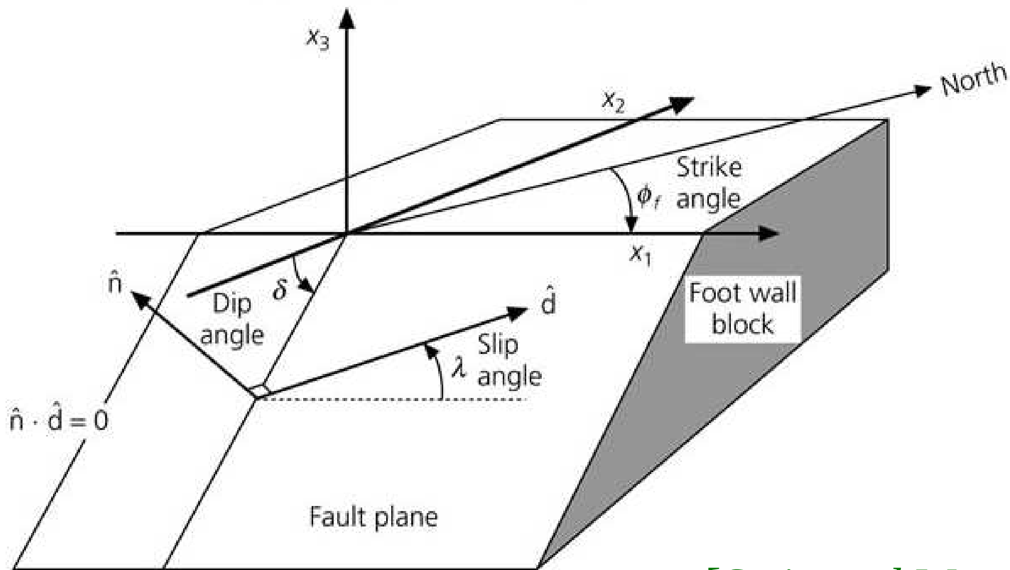
[*Stein and Wysession*, 2002]

HOW CAN WE

- Best describe this gemoetry ?
- Determine it from seismological data ?
- Represent it graphically in simple terms ?

EARTHQUAKE SOURCE GEOMETRY

THREE ANGLES are necessary to describe the focal mechanism of an earthquake:



[*Stein and Wysession*, 2002]

- The *strike angle* ϕ identifies the azimuth of the trace of the fault on the horizontal Earth surface;

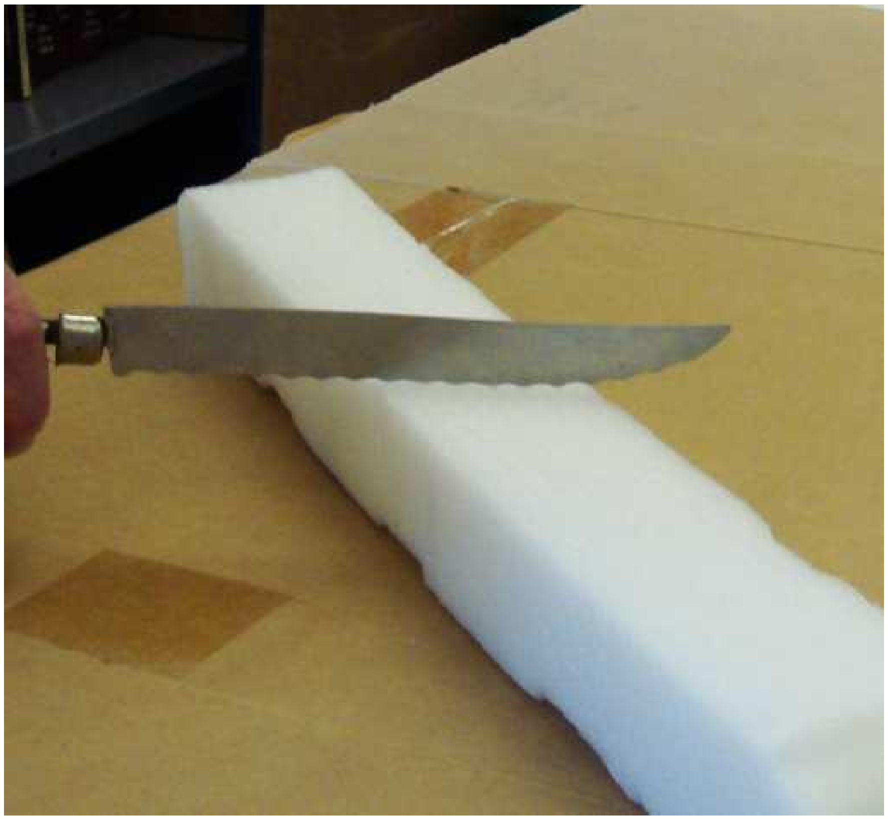
- The *dip angle* δ indicates how steeply the fault penetrates the Earth;

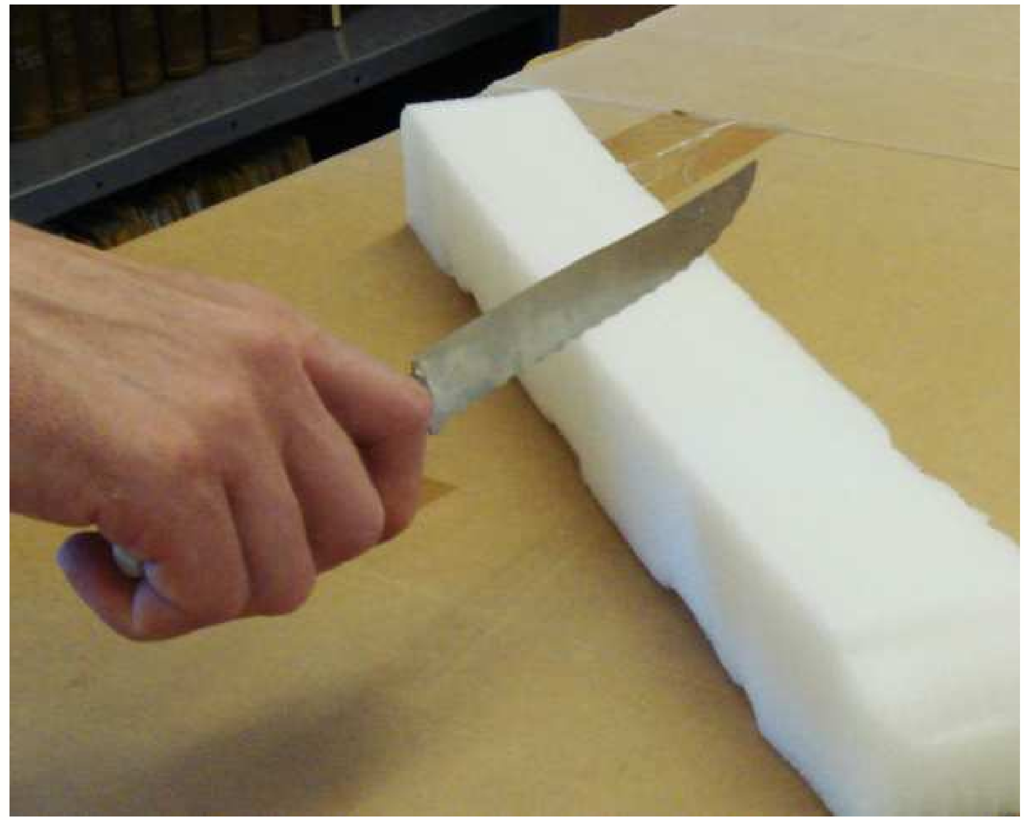
- The *slip angle* λ describes the relative motion of the two blocks on the fault plane determined by ϕ and δ.

→ The physical description of an earthquake source is thus **more complex than a vector** since it requires *three* angles as opposed to two.

The strike angle ϕ

(between 0° and 360°)





defines the azimuth of the trace of the fault on the Earth's surface

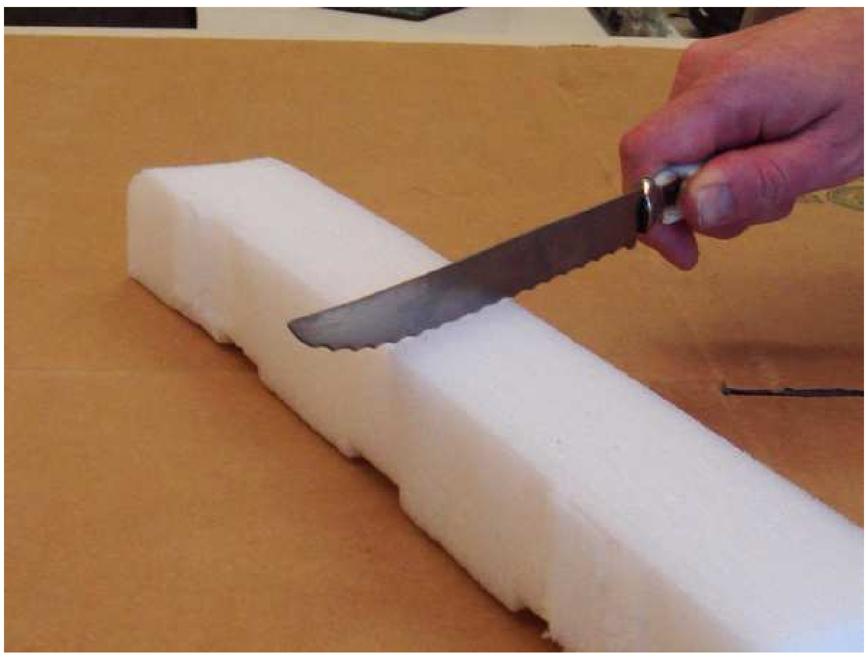
(the *orientation of the knife*)

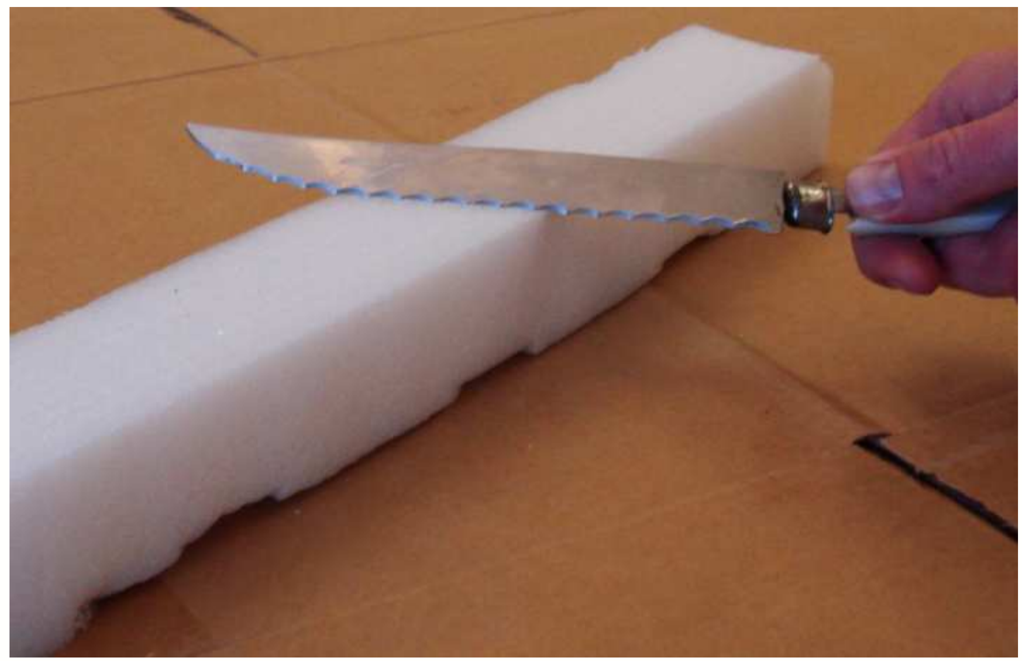
The dip angle δ

(between 0° and 90°)

defines the slope (dip) of the fault to be cut through the material

(the *inclination of the blade* on the horizontal)





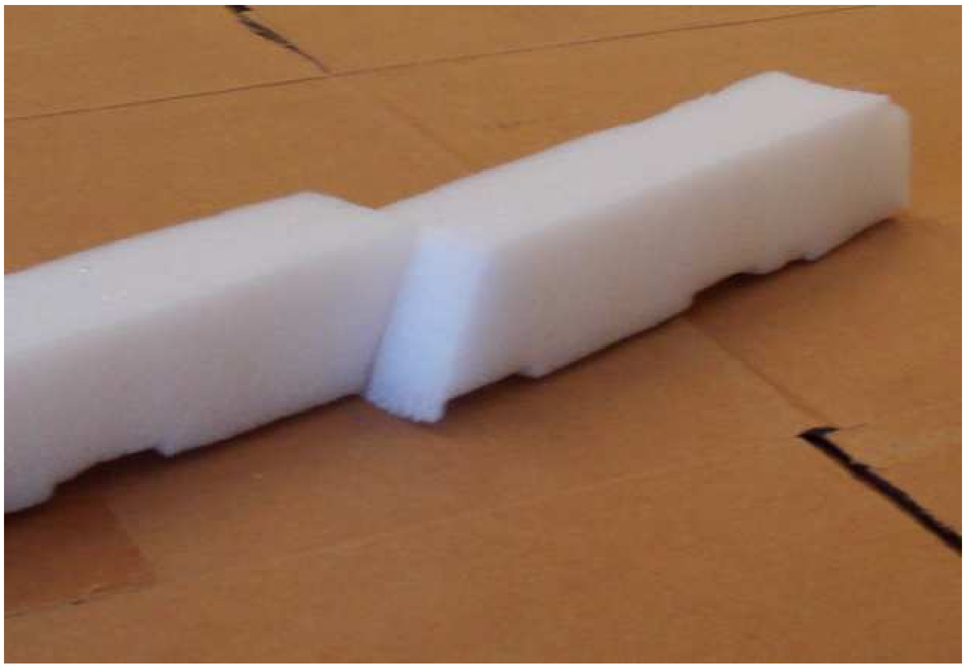
Vertical dip (δ = 90°)

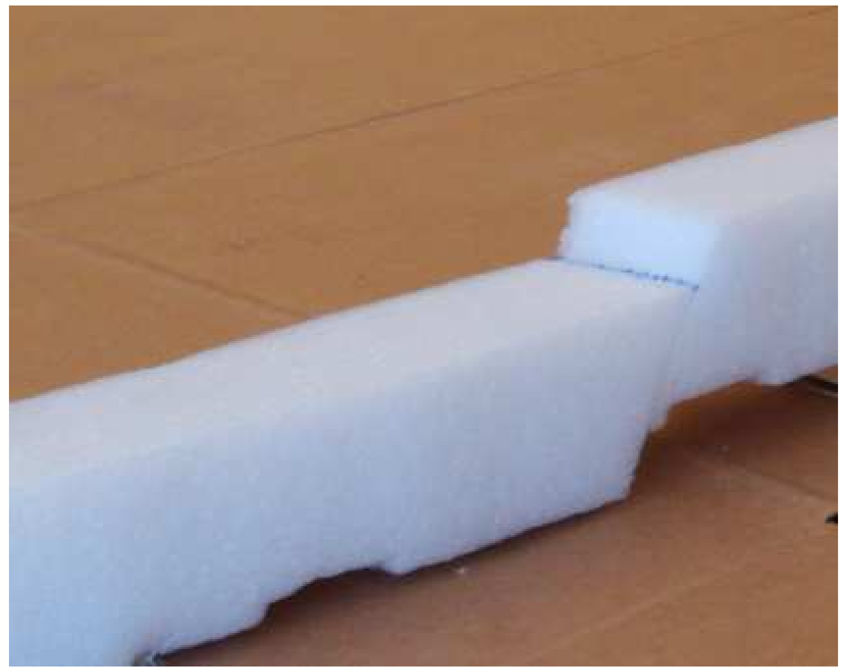
Shallow dip (δ = 30°)

The slip (or rake) angle λ

(between 0° and 360°)

defines the direction of motion of the blocks on the fault plane (cut) defined by ϕ and δ.





Strike-slip (λ = 180°)

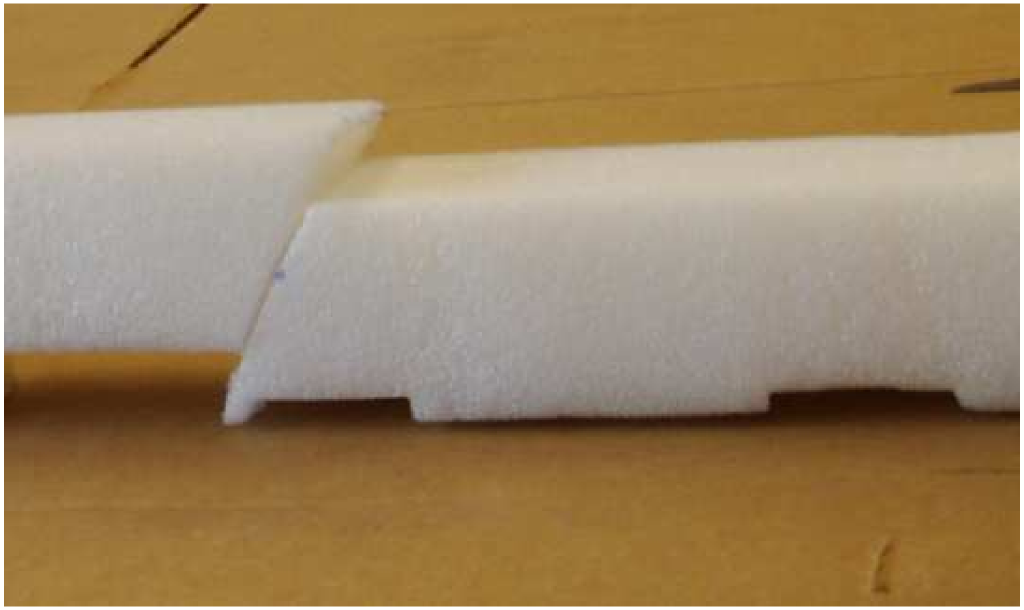
No vertical motion

Dip-slip (λ = 270°)

Motion along line of steepest descent

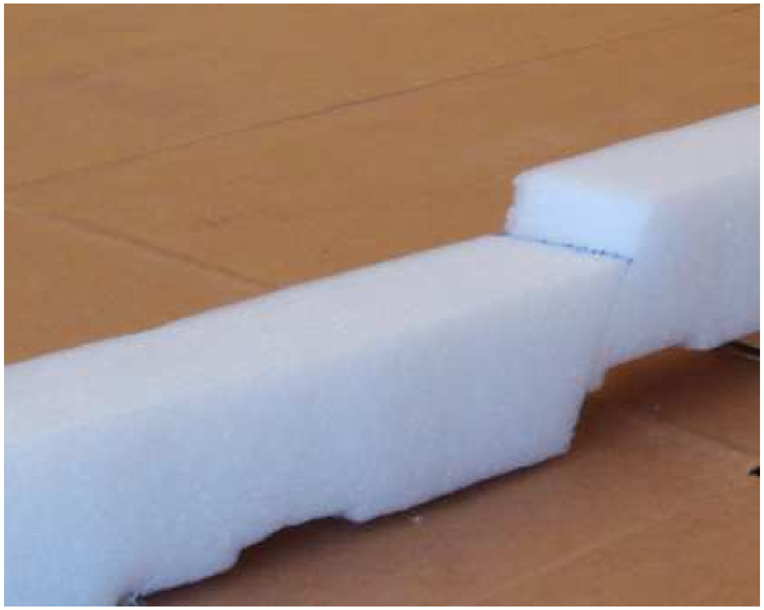
Varying the slip (or rake) angle λ (ctd.)

Thrust Faulting ($\lambda = 90°$)



(Typical of subduction zones)

Normal Faulting ($\lambda = 270°$)

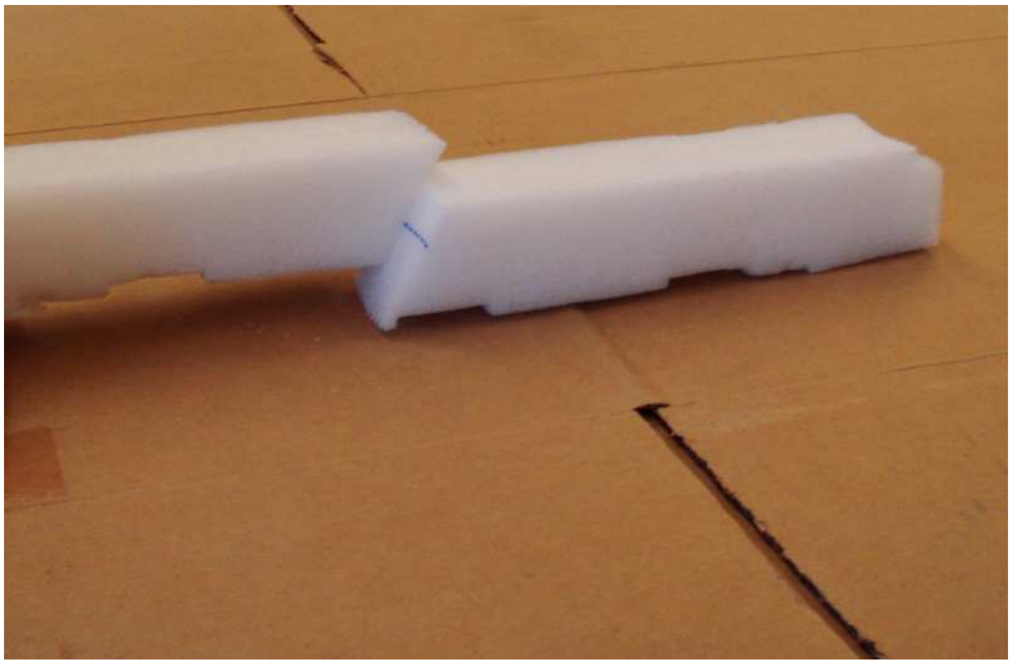


(Typical of tensional environments)

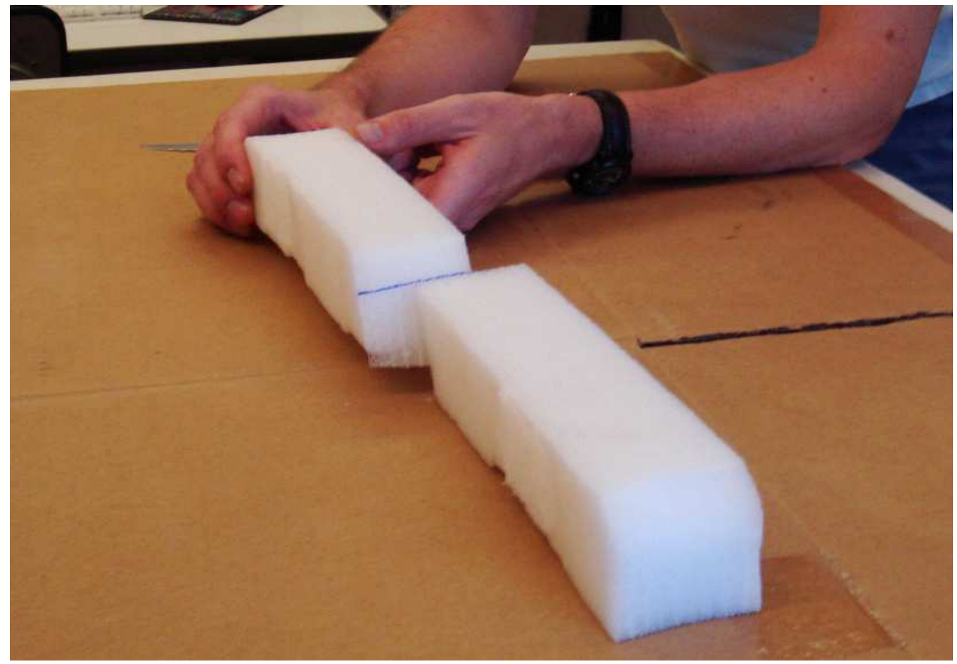
Varying the slip (or rake) angle λ (ctd.)

HYBRID MECHANISMS

Thrust and Strike-slip (λ = 120°)

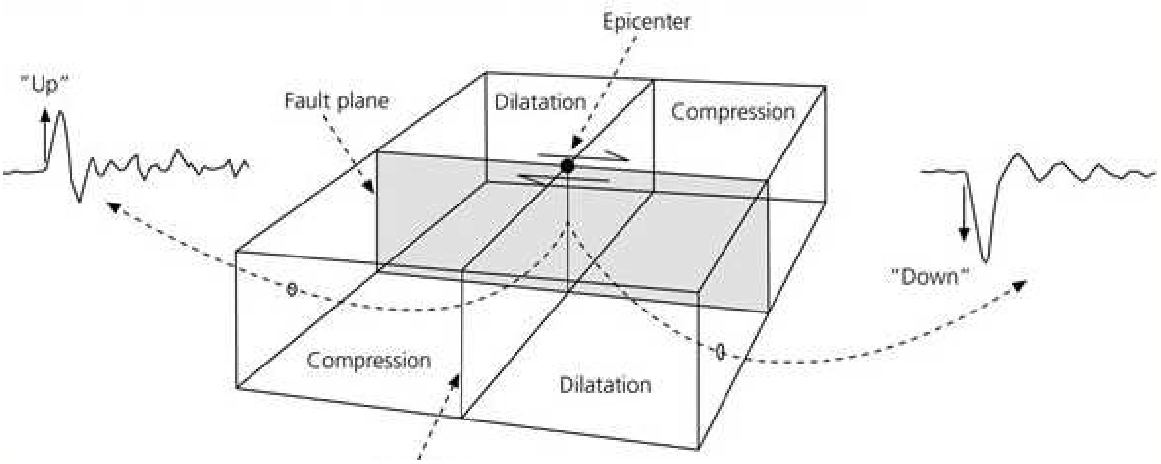


Normal Faulting and Strike-Slip (λ = 315°)



EARTHQUAKE SOURCE GEOMETRY

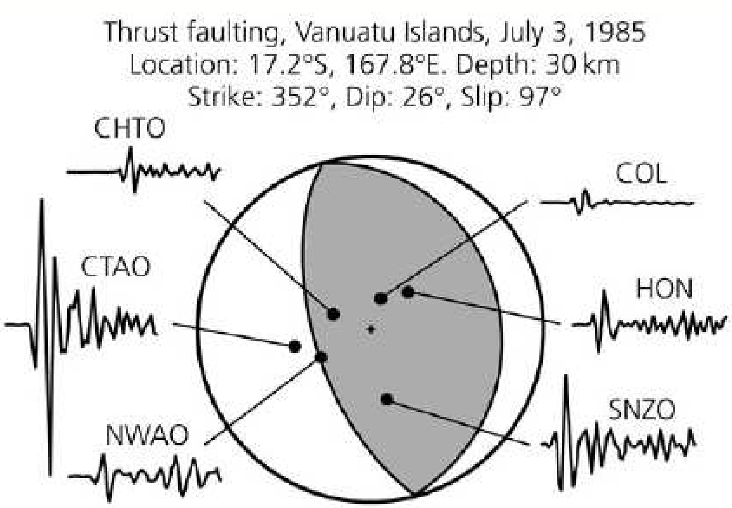
Double-Couple mechanisms give rise to *P* waves which can have positive (first-motion "up") or negative (first-motion "down") initial motions.



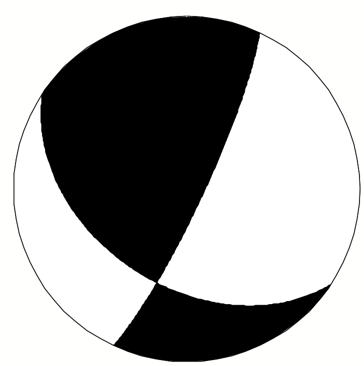
[*Stein and Wysession*, 2002]

The repartition of such motions on a small sphere surrounding the source involves four alternating *quadrants* in space.

We represent focal mechanisms by giving a stereographic view of a small focal [hemi]sphere with positive quadrants shaded and negative ones left open.

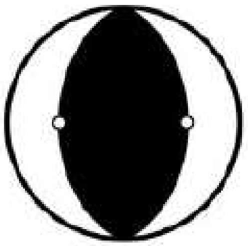


General case

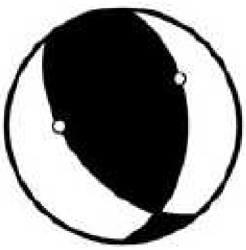


These are called *"focal beachballs"*.

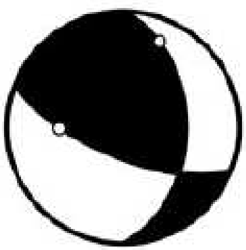
EXAMPLES of EARTHQUAKE SOURCE GEOMETRIES



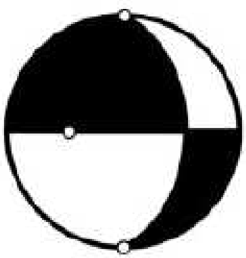
$\lambda = 90°$ — Pure dip-slip (thrust)



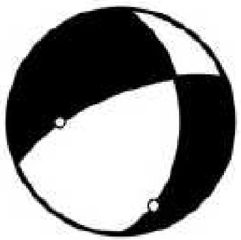
$\lambda = 120°$ — Mostly dip-slip with some strike-slip



$\lambda = 150°$ — Mostly strike-slip with some dip-slip

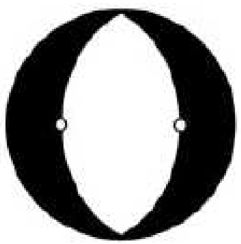


$\lambda = 180°$ — Pure strike-slip (right lateral)



$\lambda = 210°$ — Mostly strike-slip with some dip-slip

$\lambda = 240°$ — Mostly dip-slip with some strike-slip

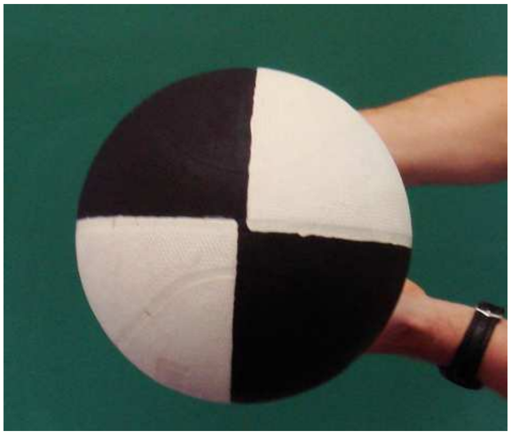


$\lambda = 270°$ — Pure dip-slip (normal)

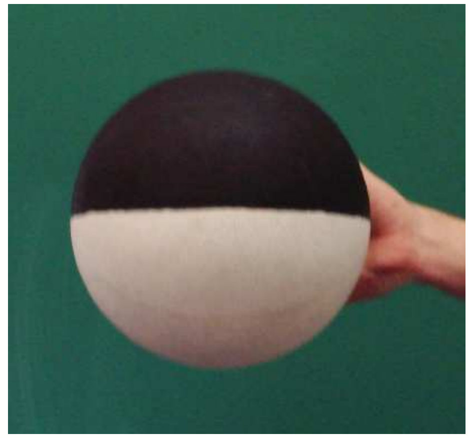
[*Stein and Wysession*, 2002]

ALL FOCAL MECHANISMS ARE CREATED EQUAL...

They are just ONE SOLID ROTATION away from Each Other



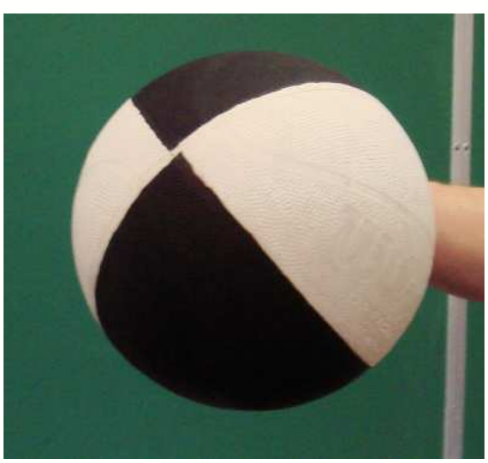
Strike-Slip



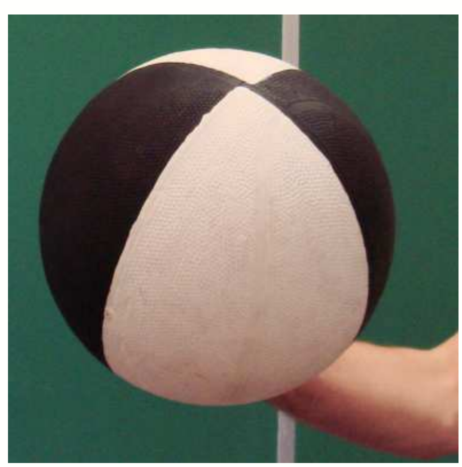
Vertical Dip-Slip

Thrust

Normal



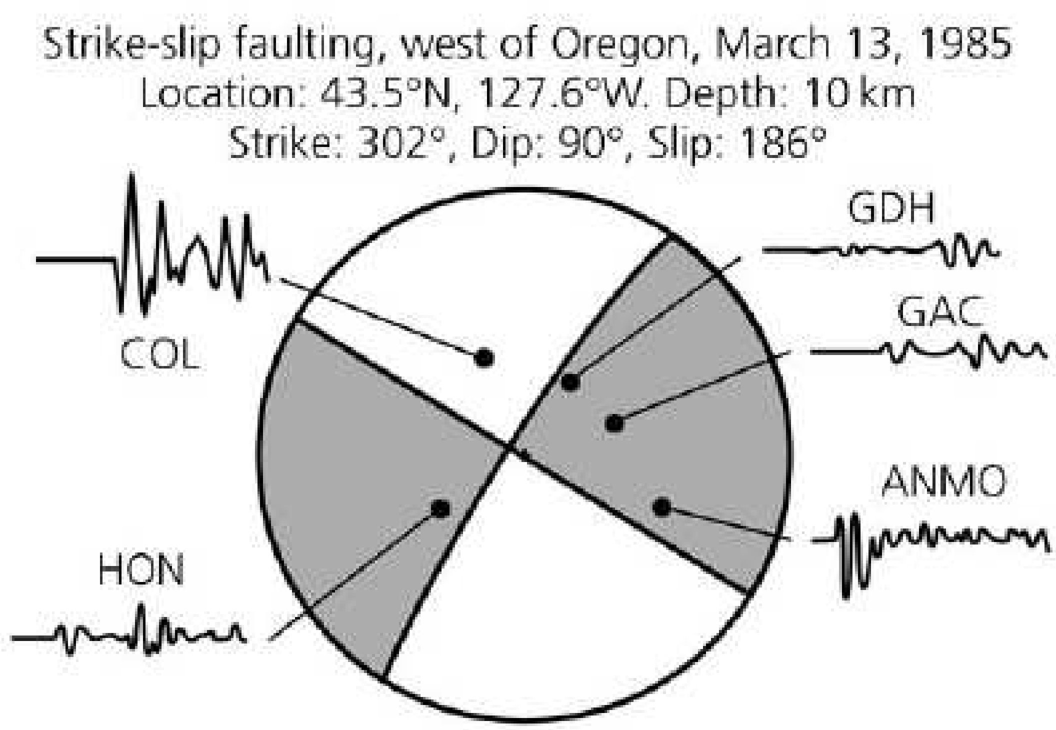
Hybrid



DETERMINATION of FOCAL MECHANISMS

- *Historically*

  Examine first motion of *P* waves and plot them on a beach ball.



[*Stein and Wysession*, 2002]

DETERMINATION of FOCAL MECHANISMS

- *Modern Technique*

  Directly invert waveforms at many stations for the *components of the moment tensor* representing the double-couple.

"Centroid Moment Tensor Inversion"

→ This is possible because that seismic ground displacement is a linear combination of these components.

Body Waves

$$u_n(\mathbf{x};t) = M_{pq} * G_{np,q} \;=\; \frac{\gamma_n \gamma_p \gamma_q}{4\pi\rho\alpha^3 r}\;\dot{M}_{pq}\left(t - \frac{r}{\alpha}\right)$$

P waves

S waves

$$-\left(\frac{\gamma_{\gamma_p} - \delta_{np}}{4\pi\rho\,\beta^3\,r}\right)\gamma_q\;\;\dot{M}_{pq}\left(t - \frac{r}{\beta}\right)$$

Normal modes

$$\mathbf{u}(r,t) = \sum_N \mathbf{s}_n(\mathbf{r})\left(\varepsilon_n^*(\mathbf{r}_s) : \boldsymbol{M}(\mathbf{r}_s)\right)\cdot\;\frac{1-\cos\omega_n t\,\exp(-\omega_n t/2Q_n)}{\omega_n^2}$$

NOTE LINEARITY of all Equations with respect to M_{pq} .

NEAR-REAL TIME CMT SOLUTIONS

Computed routinely by the NEIC (body waves) and the Global CMT (*ex*-Harvard) project (body *and surface* waves)

Example: **08 JUL 2007, ALEUTIAN ISLANDS**

NEIC

```
   07/08/02 03:21:46.54
ANDREANOF ISLANDS, ALEUTIAN IS.
Epicenter:  51.358 -179.929
MW 6.6

USGS MOMENT TENSOR SOLUTION
Depth  21         No. of sta: 54
Moment Tensor;   Scale 10**19 Nm
  Mrr= 0.73       Mtt=-0.59
  Mpp=-0.14       Mrt= 0.53
  Mrp= 0.41       Mtp=-0.40
 Principal axes:
  T  Val=  0.99  Plg=69  Azm=316
  N        0.09      4       57
  P       -1.07     20      148

Best Double Couple:Mo=1.0*10**19
 NP1:Strike=246 Dip=25 Slip= 100
 NP2:         55     65        85

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    -------################--
  ------#######################
 -----#######################---
 ---########   ##############----
---######### T ############------
--##########   ##########--------
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-##################--------------
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 ############-------------------
  #####---------------   ------
    ------------------ P ----
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```
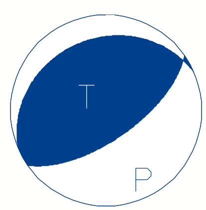

$M_0 = 1.0 \times 10^{26}$ dyn*cm



Global CMT

```
August 2, 2007, ANDREANOF ISLANDS, ALEUTIAN IS., MW=6.7

Goran Ekstrom

CENTROID-MOMENT-TENSOR  SOLUTION
GCMT EVENT:     C200708020321A
DATA: II IU CU IC GE
L.P.BODY WAVES: 64S, 165C, T= 50
MANTLE WAVES:   62S, 138C, T=125
SURFACE WAVES:  64S, 172C, T= 50
TIMESTAMP:      Q-20070802104412
CENTROID LOCATION:
ORIGIN TIME:      03:21:51.2 0.1
LAT:51.11N 0.00;LON:179.66W 0.01
DEP: 32.2  0.2;TRIANG HDUR:  5.6
MOMENT TENSOR: SCALE 10**26 D-CM
RR= 1.010 0.006; TT=-1.050 0.005
PP= 0.031 0.005; RT= 0.740 0.010
RP= 0.716 0.010; TP=-0.403 0.004
PRINCIPAL AXES:
1.(T) VAL=  1.484;PLG=64;AZM=297
2.(N)       0.045;     15;     60
3.(P)      -1.538;     21;    156
BEST DBLE.COUPLE:M0= 1.51*10**26
NP1: STRIKE=271;DIP=27;SLIP= 123
NP2: STRIKE= 54;DIP=67;SLIP=  74

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     ---------#####---------
   -----#################-----
  ---#######################-##
 --#############################
 -########   ##############----#
-######### T #############------#
##########   ###########---------
######################----------
####################-------------
 #################--------------
 ##############-----------------
  #########--------------------
   -----------------   -------
     --------------- P -----
       -------------   ---
           -----------
```
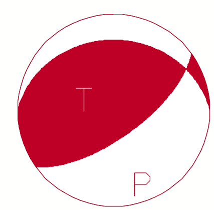

$M_0 = 1.5 \times 10^{26}$ dyn*cm



Note difference in moment

The two focal solutions are separated by a solid rotation of **11°**.