Earthquake Location

by Annabel Kelly

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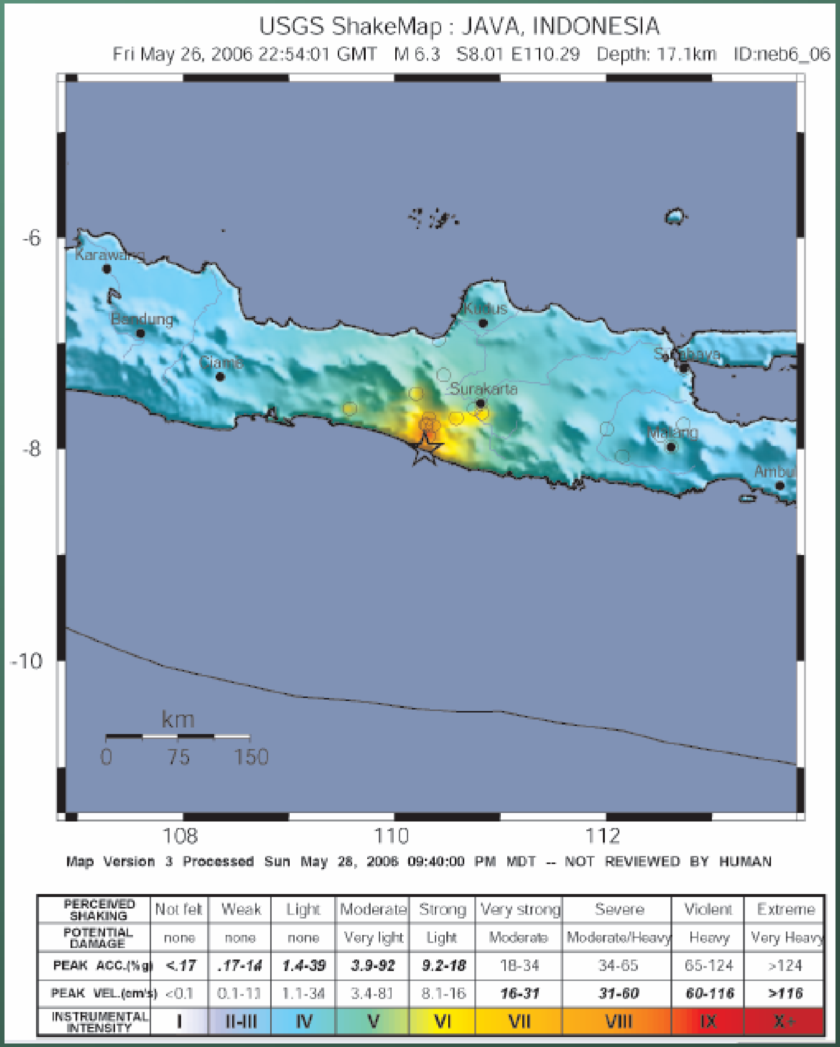
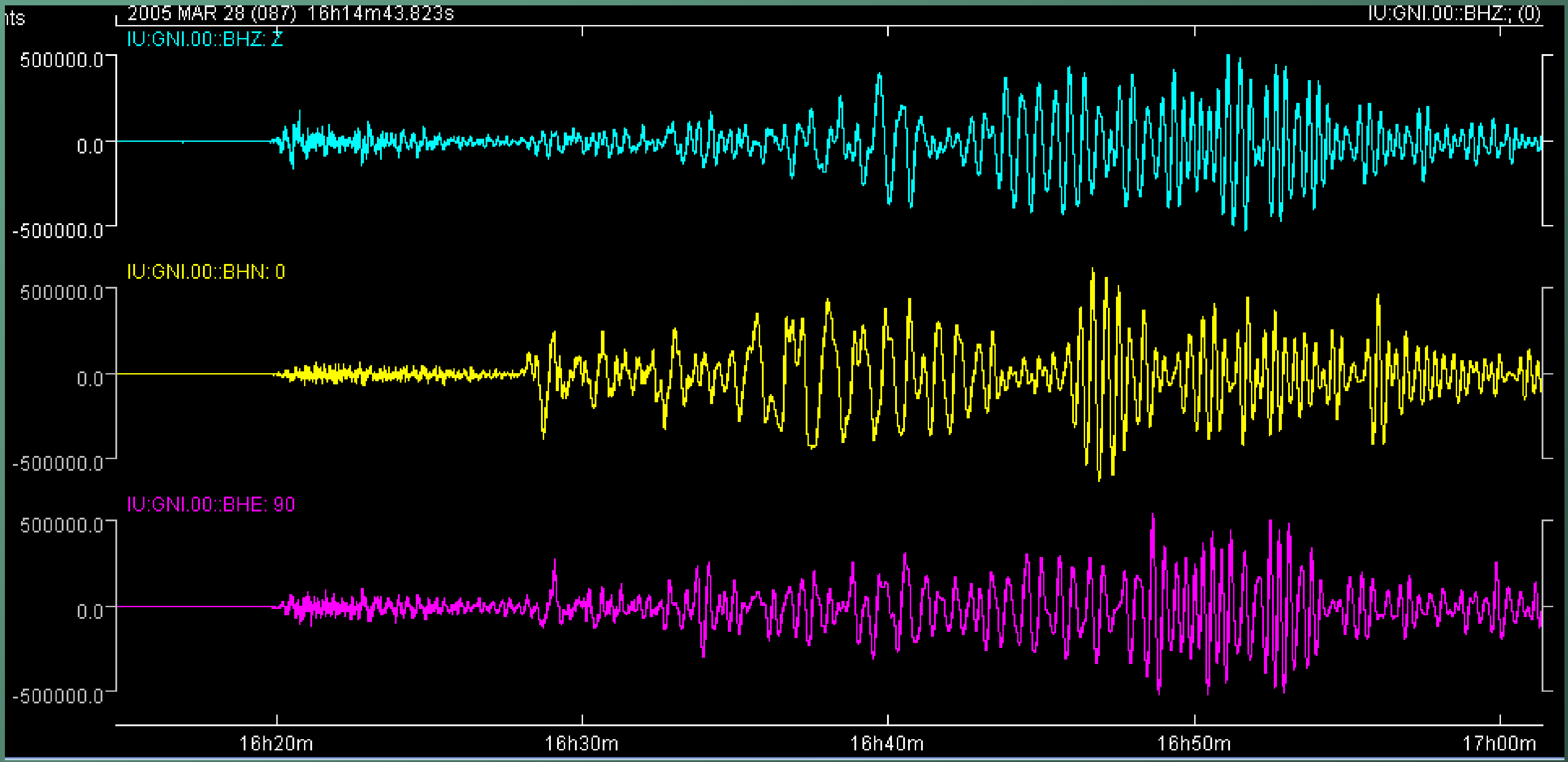
Overview

- The basic principles
  - S-P location (manual)
  - location by inversion
  - single station location
  - depth assessment
  - velocity models
- Relocation methods
  - joint hypocentral location
  - master event location
- Other related topics
  - Waveform modeling
  - Automated phase picking



Basic Principles

- 4 unknowns - origin time, x, y, z
- Data from seismograms – phase arrival times



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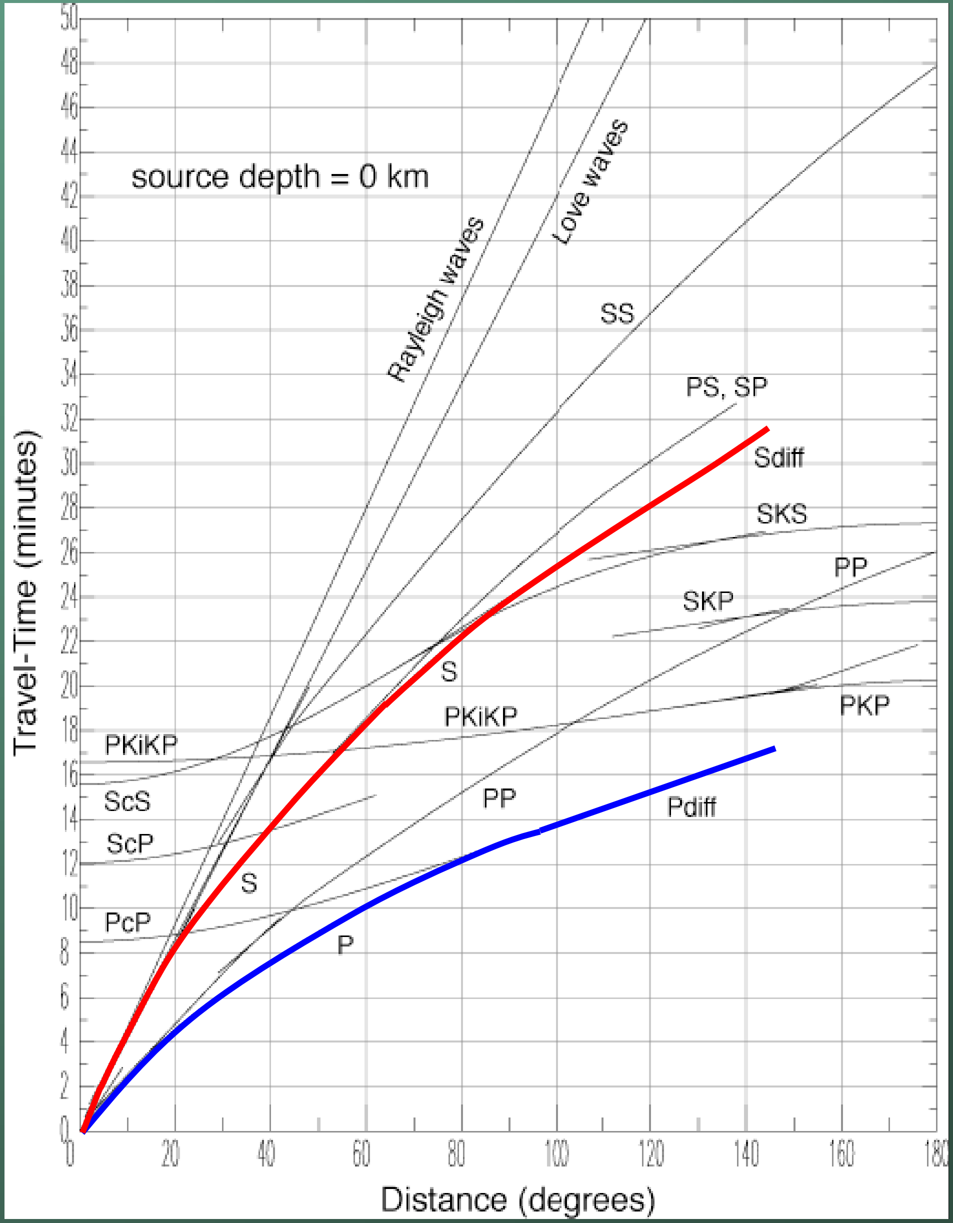
March 28, 2005 M8.7 Sumatra earthquake, as recorded at GNI station in Armenia (60 Degrees from the epicenter)

S-P time

- **Time between P and S arrivals increases with distance from the focus.**
  - **A single trace can therefore give the origin time and distance (but not azimuth)**

$$T_S - T_P = D\left(\frac{1}{V_S} - \frac{1}{V_P}\right)$$

approximates to $D = 8\left(T_S - T_P\right)$



PREM model, Dziewonski & Anderson, 1981

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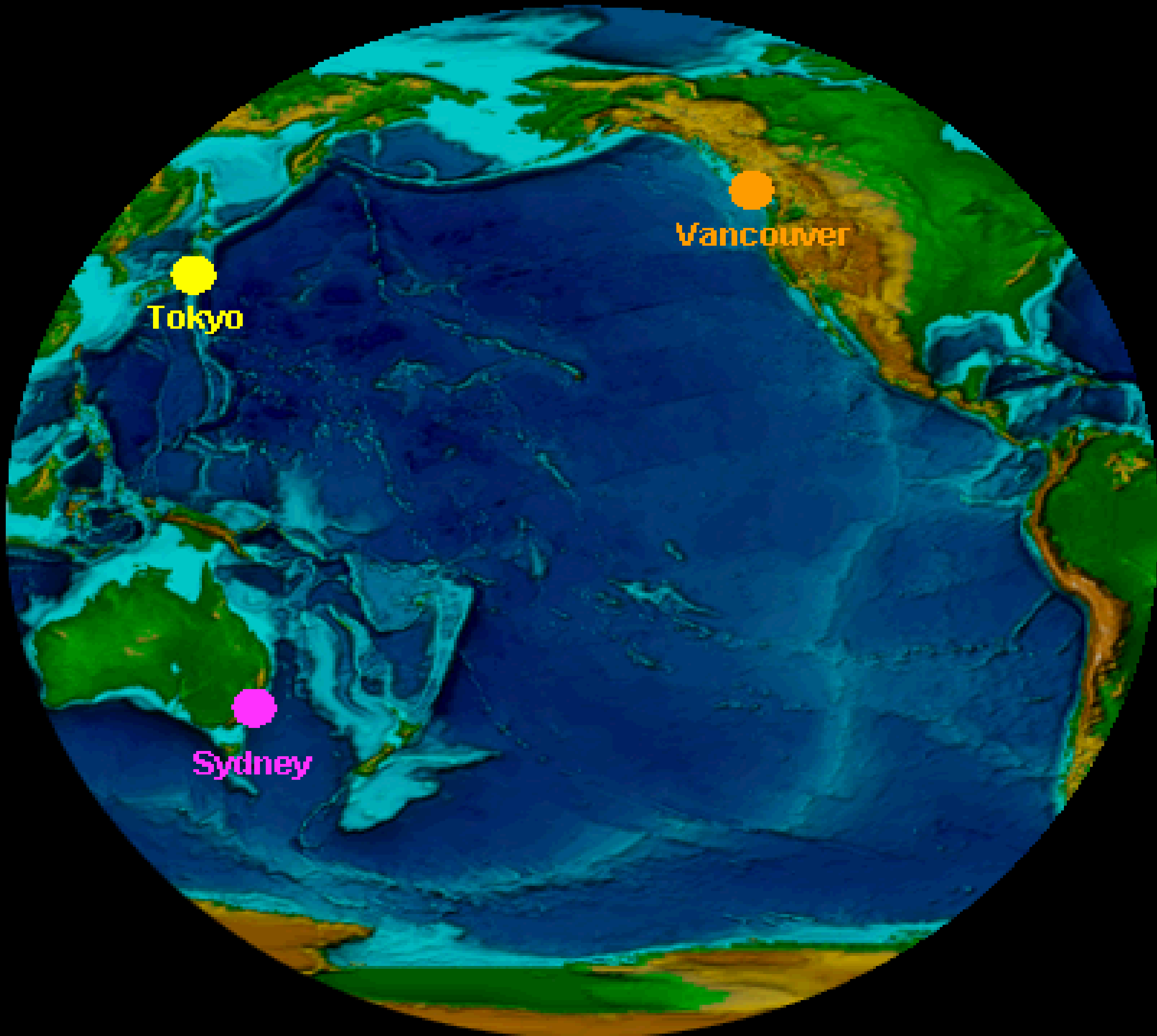
Earthquakes



AUDION

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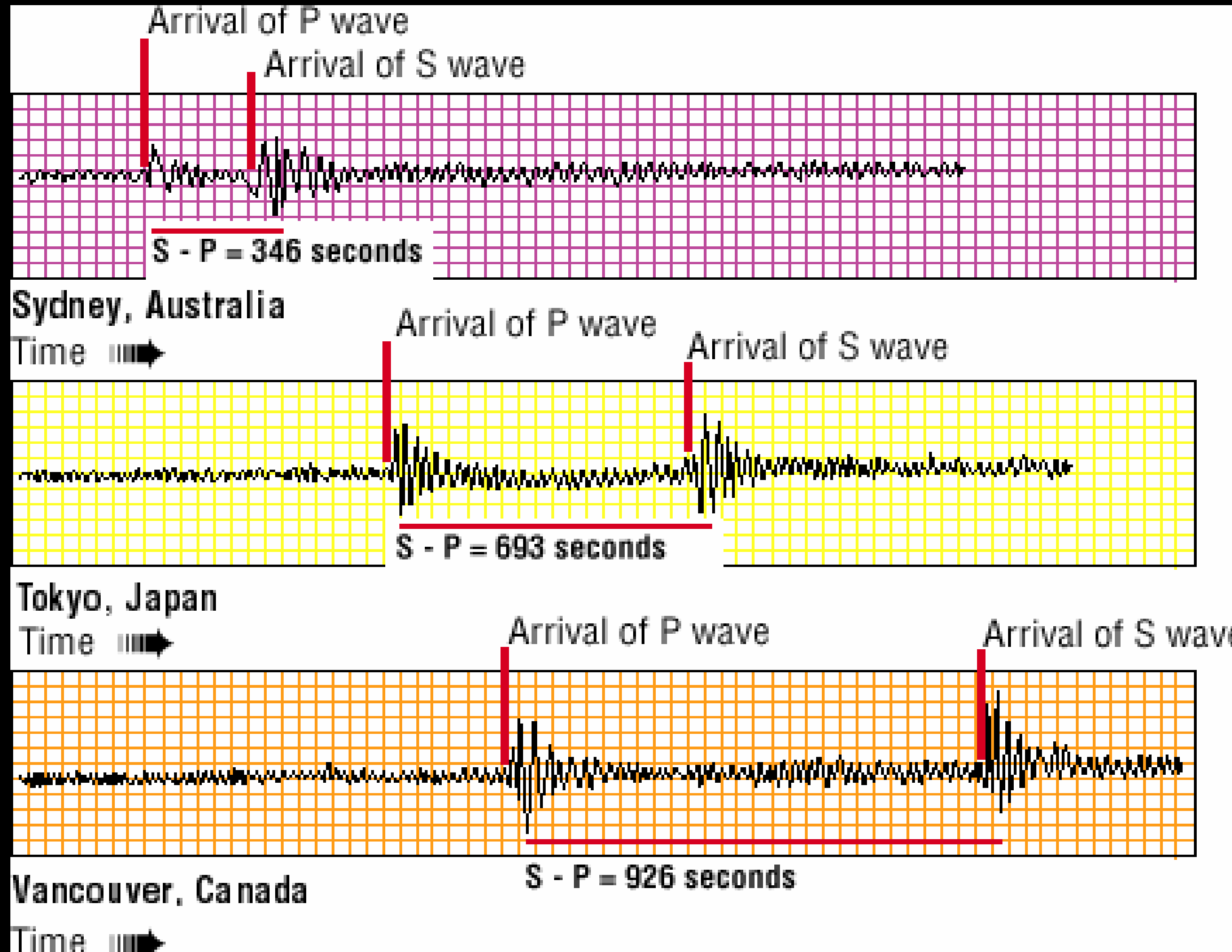
Courtesy of Dr. Qamar-uz-Zaman Chaudhary
Pakistan Mteorological Dept.

Earthquakes

Here we have measured the time intervals between the arrival of the P and the S waves at each of our three seismic locations:

	S-P interval
Sydney	346 s
Tokyo	693 s
Vancouver	926 s

Since each second of interval corresponds to about 8.4 kilometers, we calculate the distance to the epicenter from each of our seismic stations to be:



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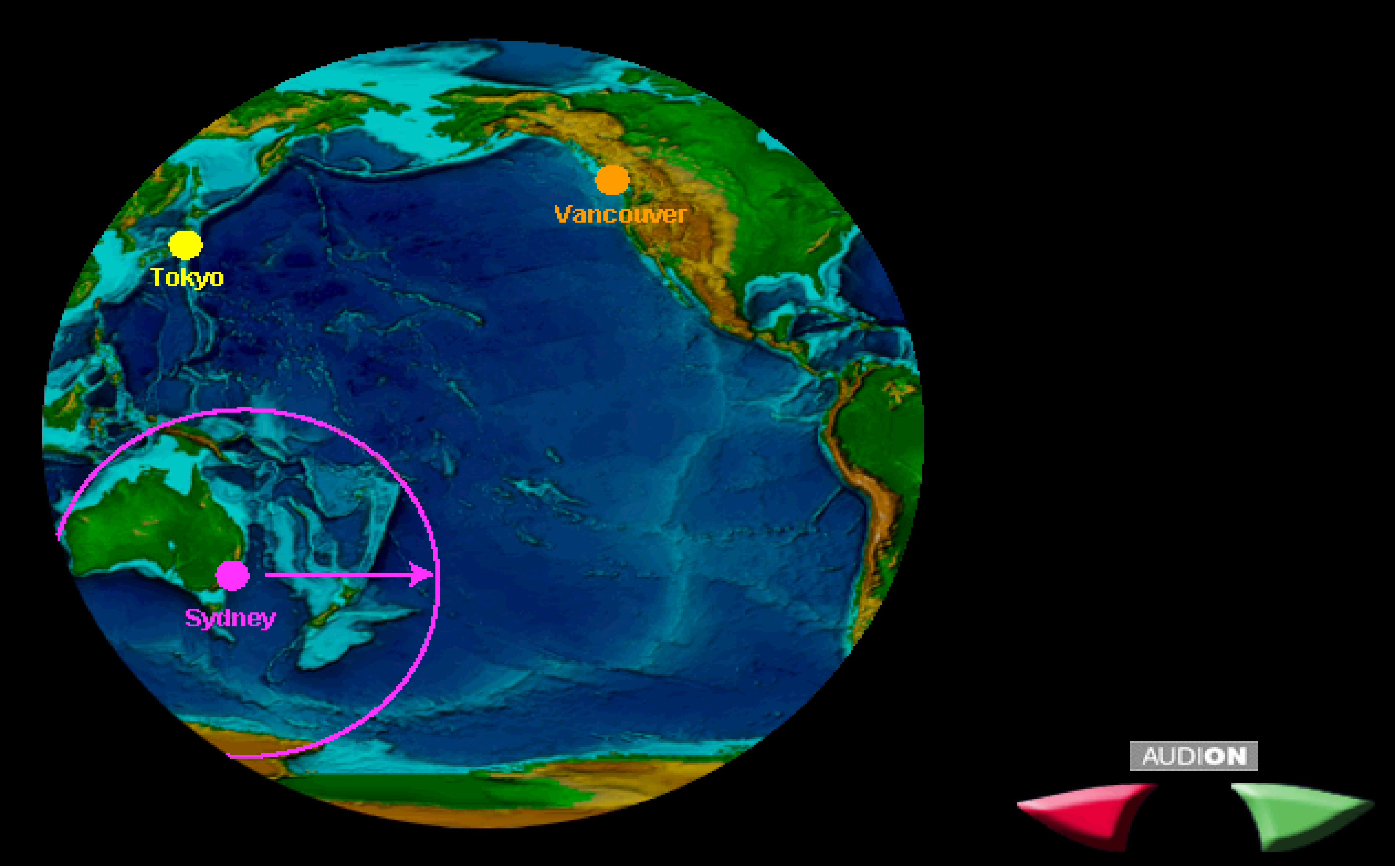
Earthquakes



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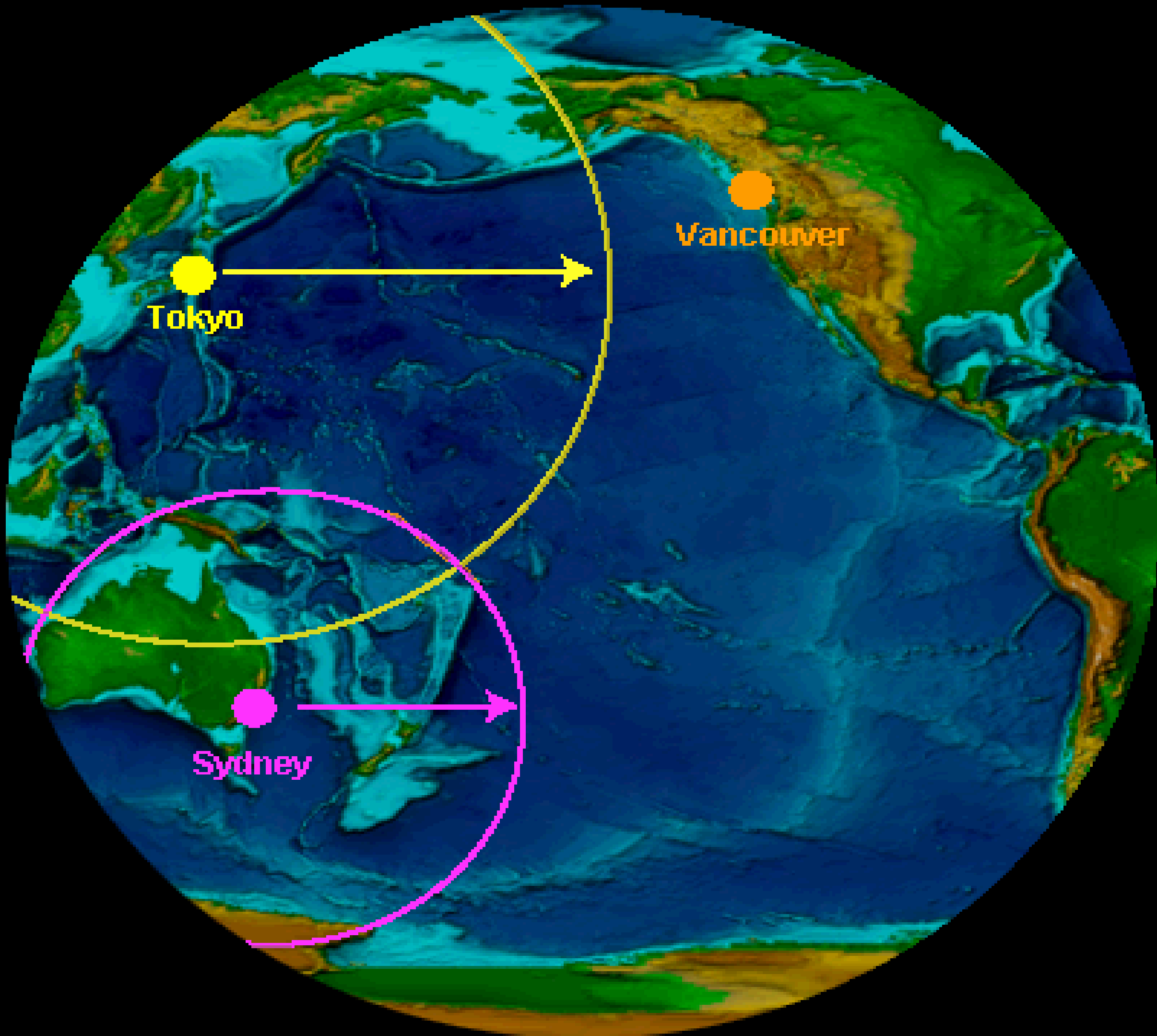
Earthquakes



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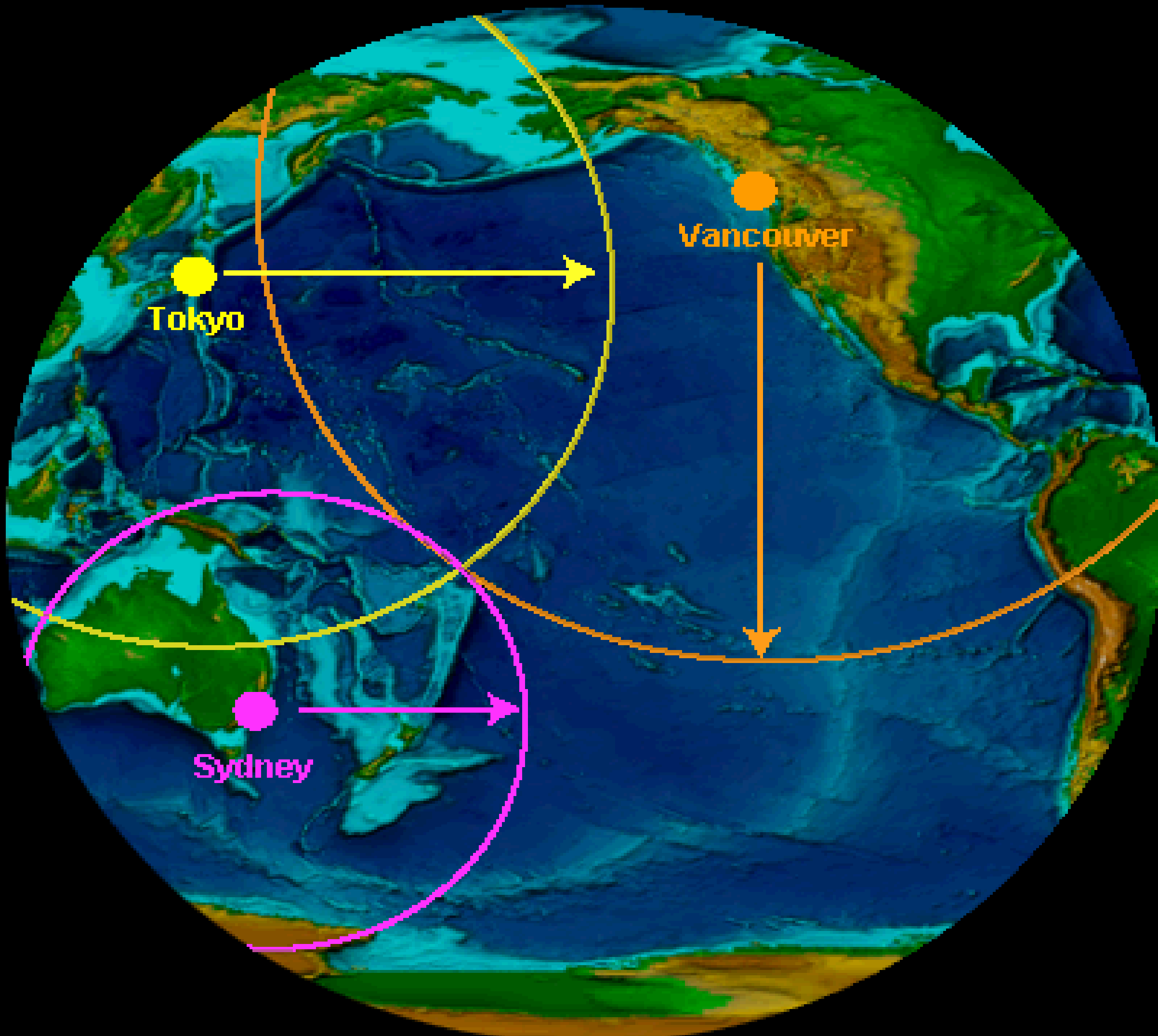
Earthquakes



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Courtesy of Dr. Qamar-uz-Zaman Chaudhary
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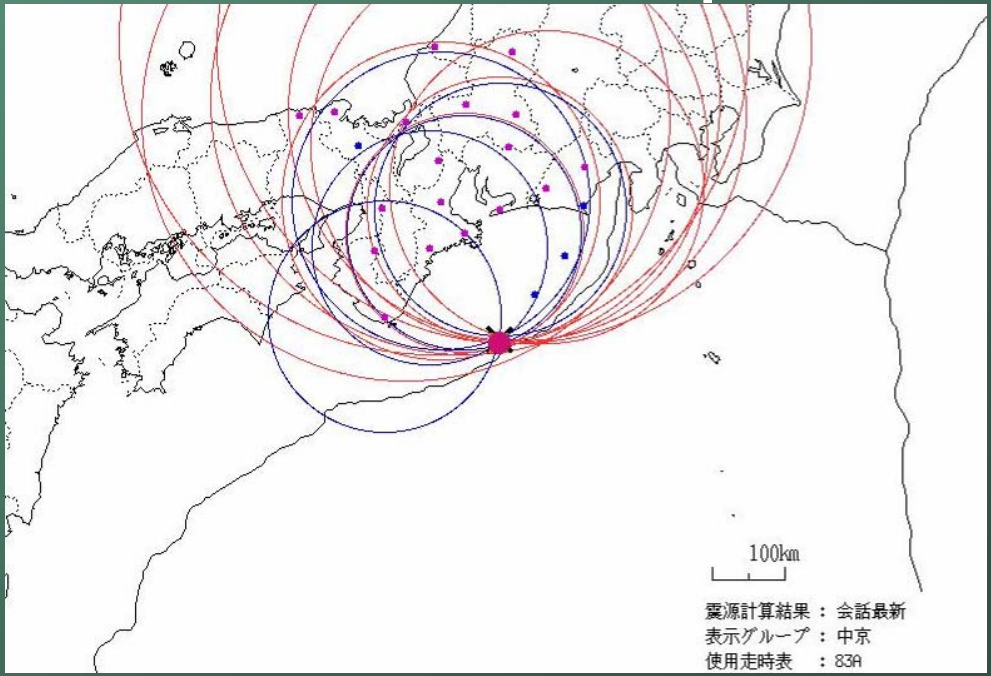
S-P method

- 1 station – know the distance - a circle of possible location
- 2 stations – two circles that will intersect at two locations
- 3 stations – 3 circles, one intersection = unique location

4+ stations – over determined problem – can get an estimation of errors

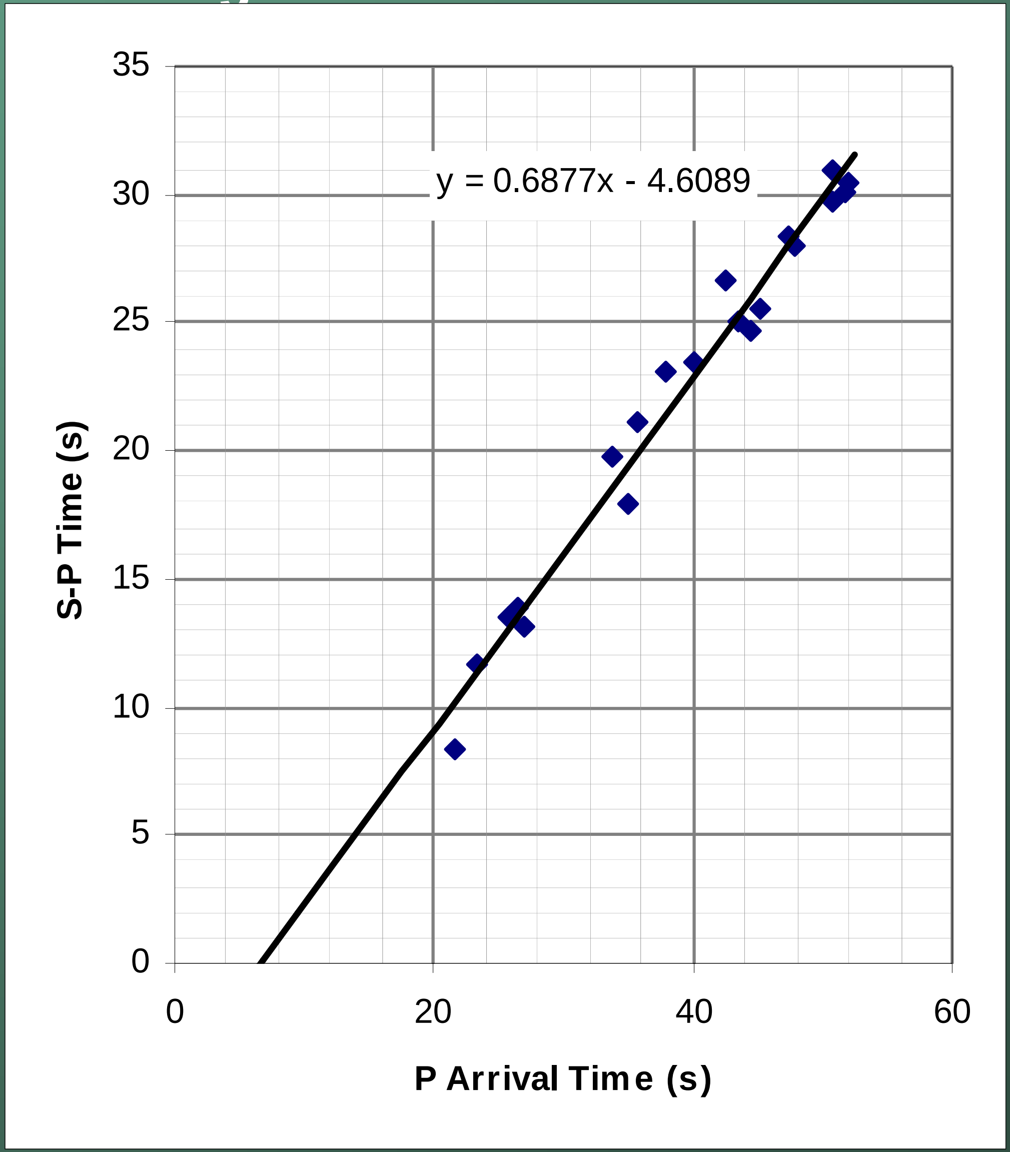


Source: Japan Meteorological Agency

Wadati diagram

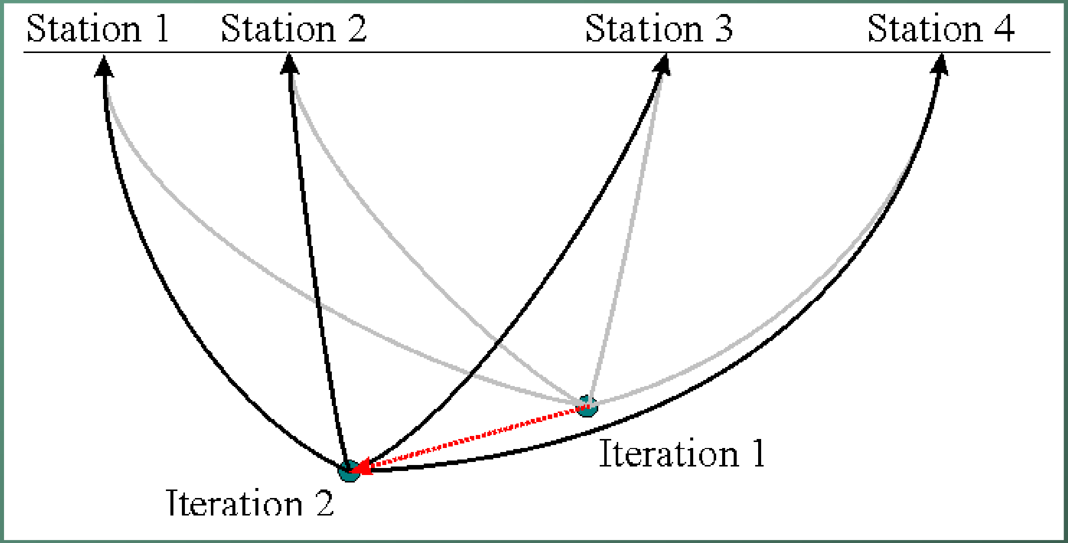
- gives the origin time (where S-TP time = 0)
- Determines Vp/Vs (assuming it's constant and the P and S phases are the same type – e.g. Pn and Sn, or Pg and Sg)
- indicates pick errors

S-P time against absolute P arrival time



Locating with P only

- The location has 4 unknowns (t,x,y,z) so with 4+ P arrivals this can be solved.



- The P arrival time has a non-linear relationship to the location, even in the simplest case when we assume constant velocity – therefore can only be solved numerically

Numerical methods

- **Calculated travel time:**

$$t^c{}_i = T(x_i, y_i, z_i, x_0, y_0, z_0) + t_0$$

- **Simplest possible relation between travel time and location:**

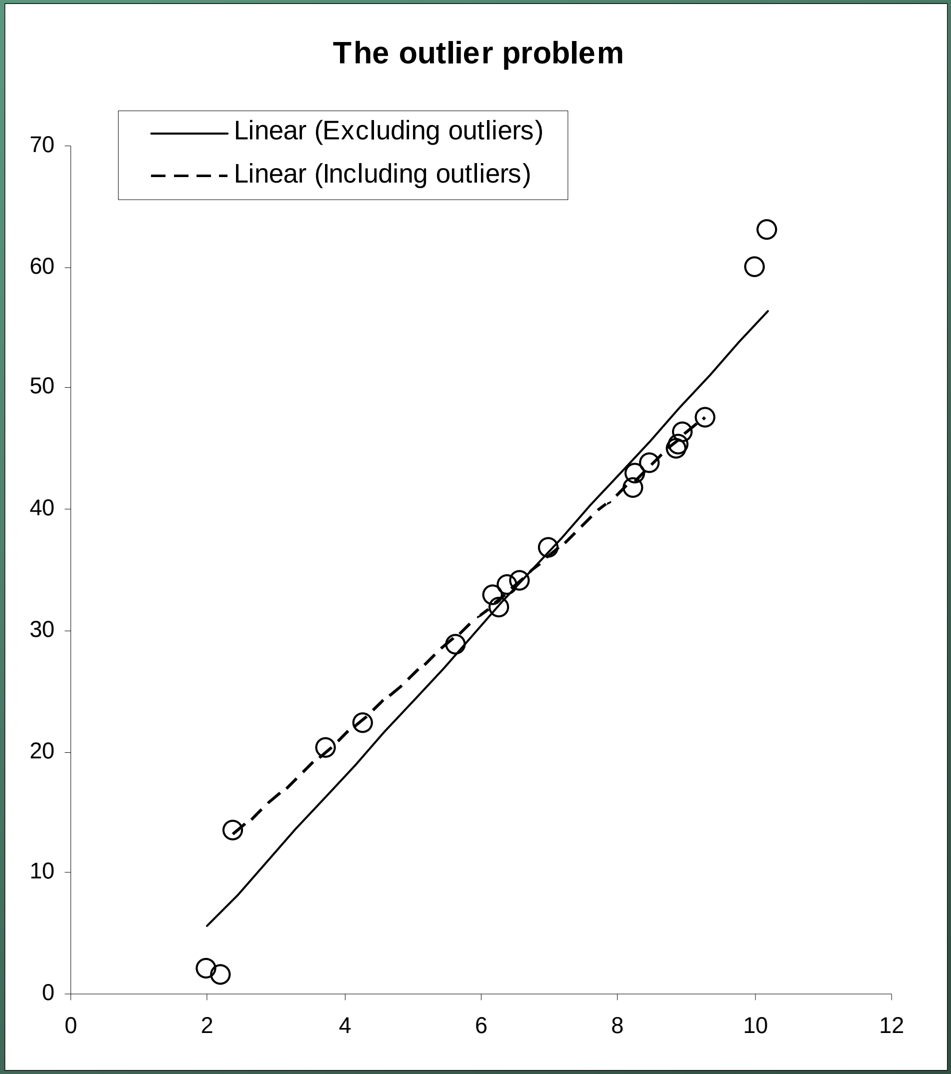
$$t_i = \frac{\sqrt{(x_0 - x_i)^2 + (y_0 - y_i)^2}}{v}$$

- **Find location by minimizing the summed residual (e):**

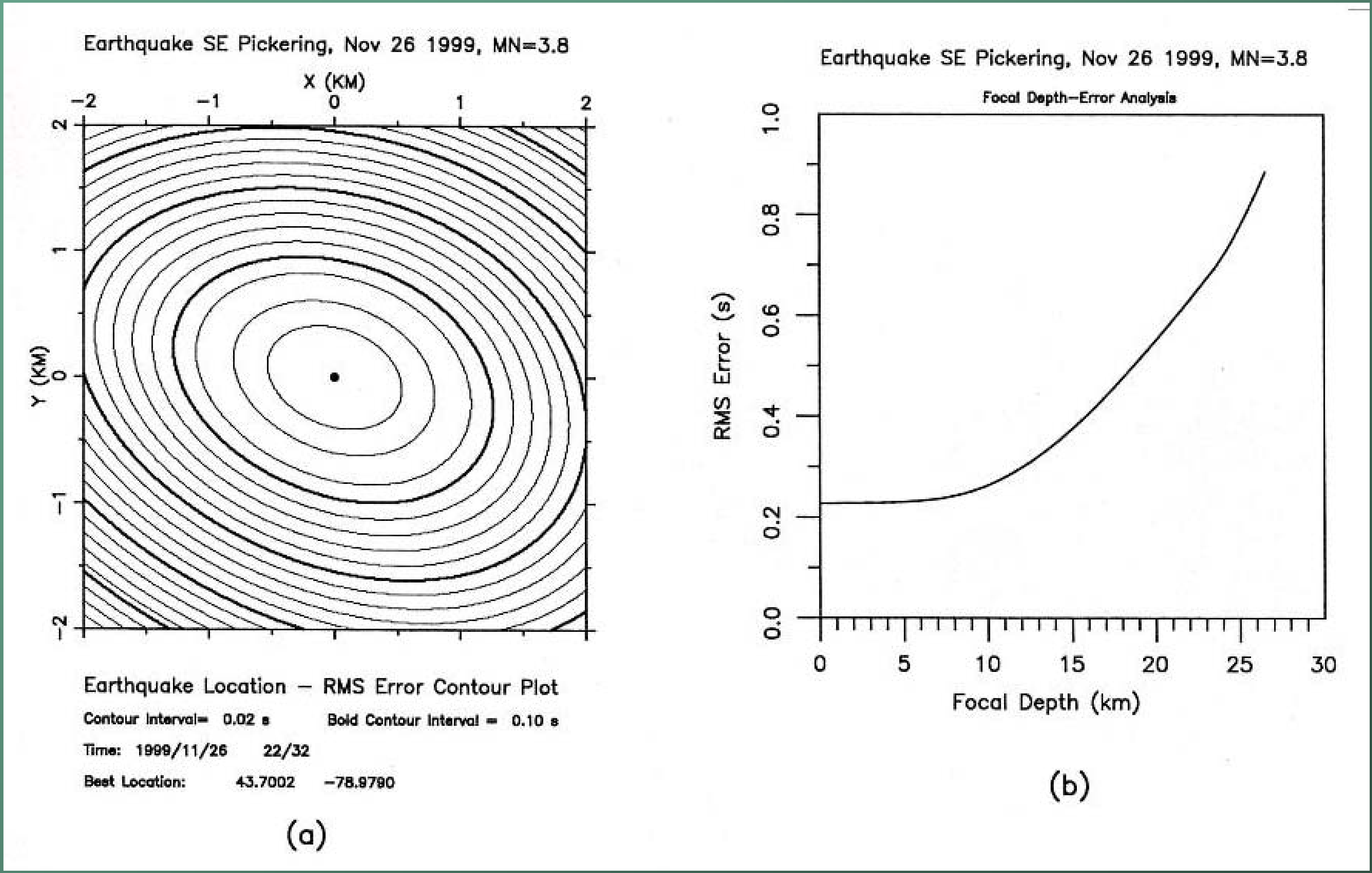
$$r_i = t_i - t^c{}_i \qquad e = \sum_{i=1}^{n} (r_i)^2$$

Least squares – the outlier problem

- **The squaring makes the solution very sensitive to outliers.**
- **Algorithms normally leave out points with large residuals**



Numerical methods – grid search



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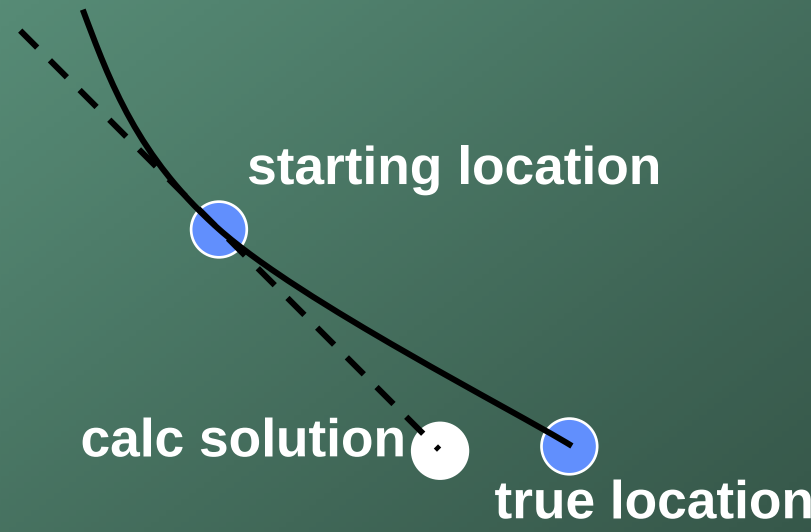
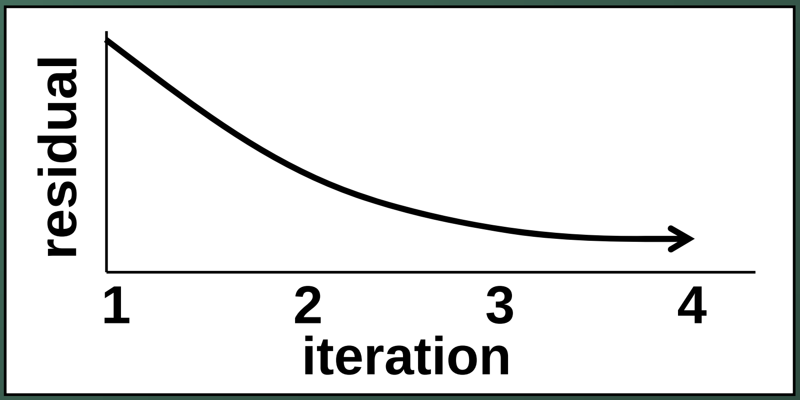
courtesy of Robert Mereu

solving using linearization

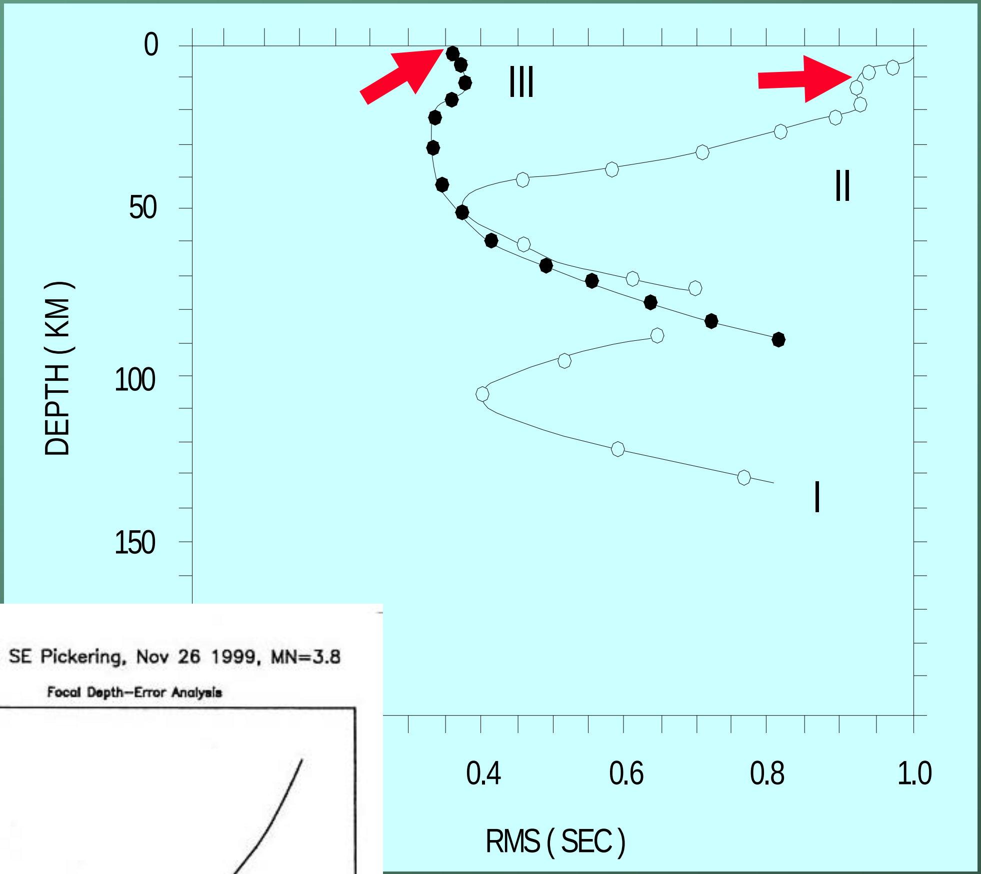
- It’s possible to solve directly using math:
  - Assume a starting location
  - Assume that the change needed is small enough that is can be considered a linear change

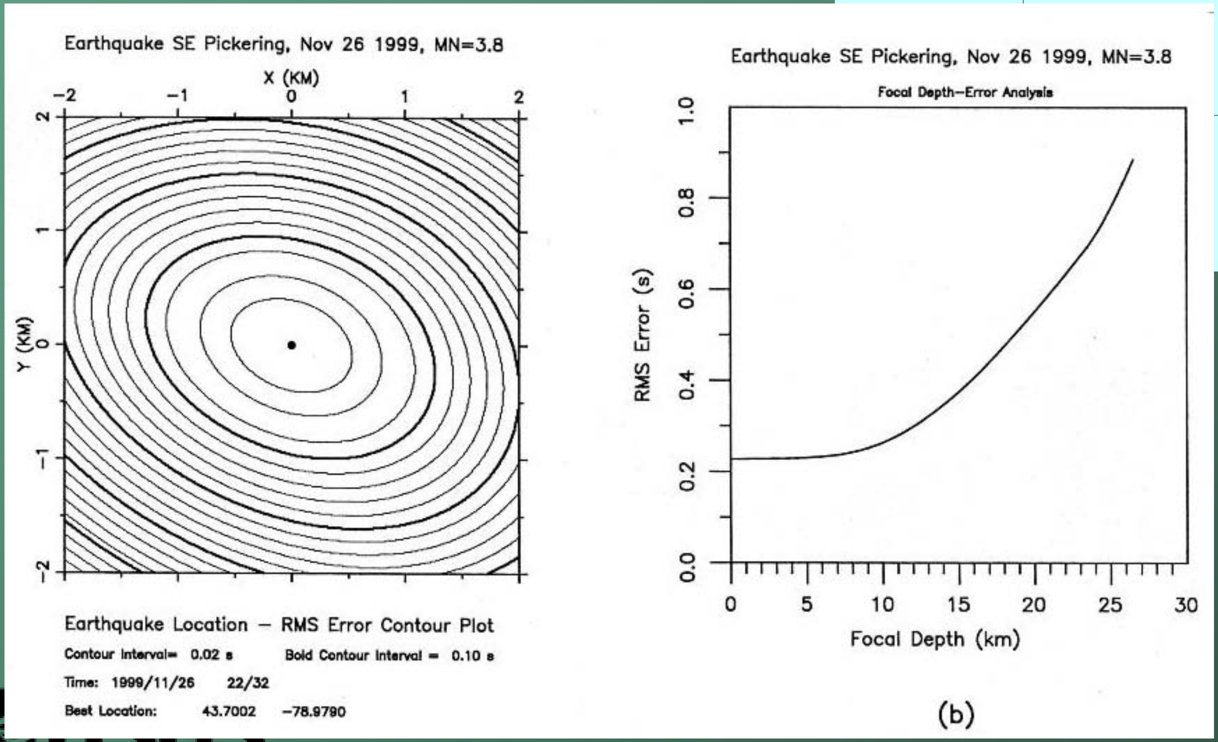


- Counteract the approximation of linearizing the problem by taking the solution as a new starting model.



- The residuals are not always a well behaved function, can have local minima



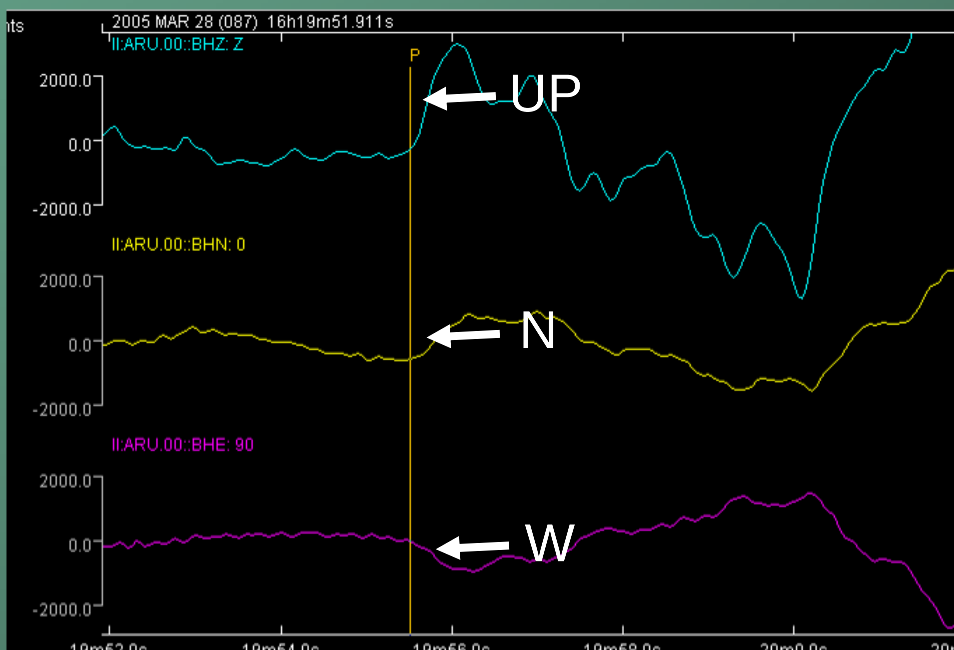


A grid search may show if there is a better solution

courtesy of Robert Mereu

Single station method

- The S-P time give the distance to the epicenter
- The ratio of movement on the horizontal components gives the azimuth



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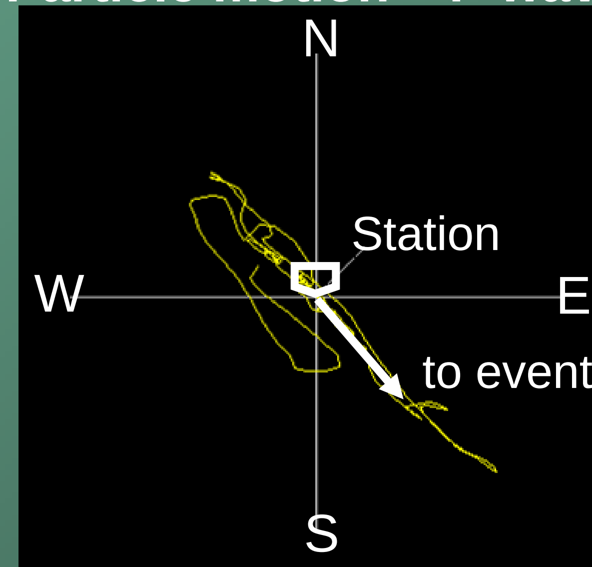
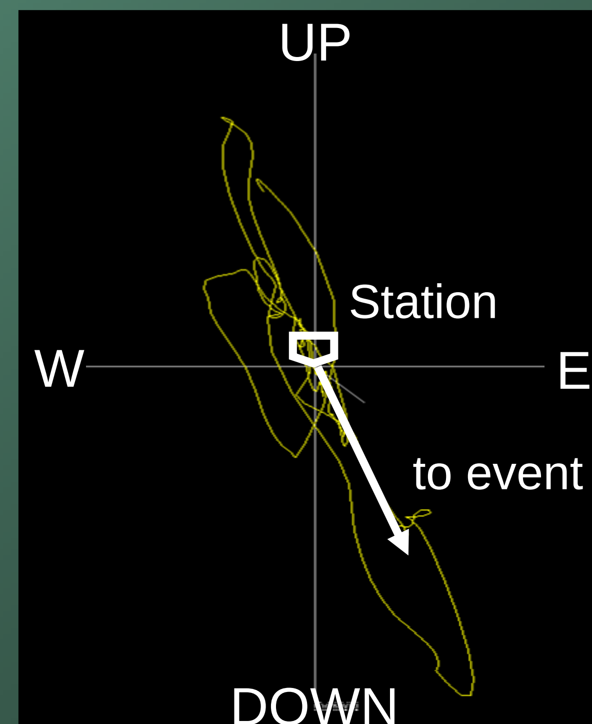
March 28, 2005 M8.7 Sumatra earthquake, as recorded at ARU station in Russia (62 Degrees from the epicenter)

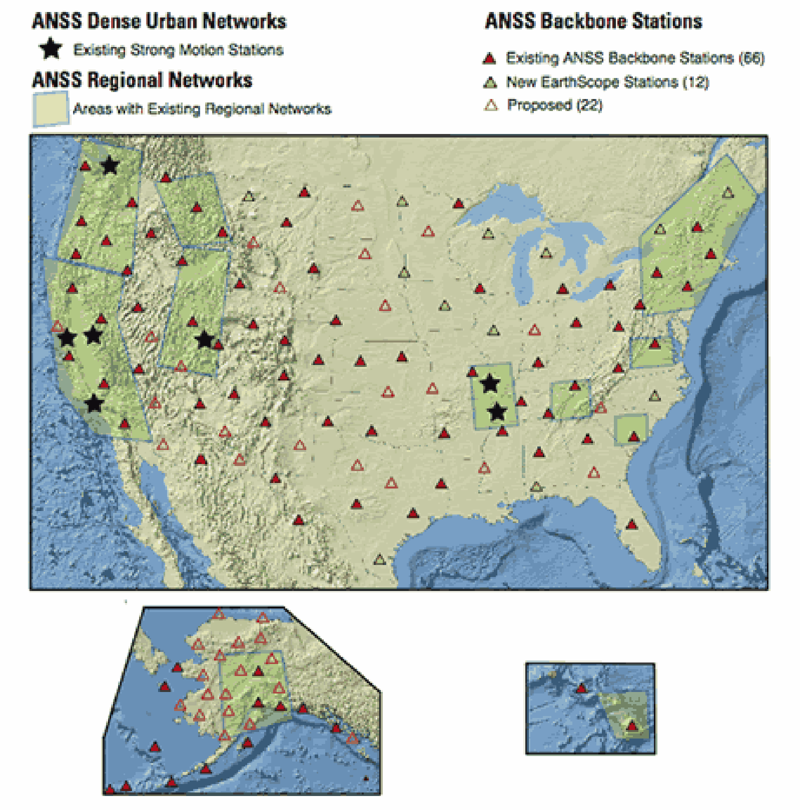
Particle motion – P wave



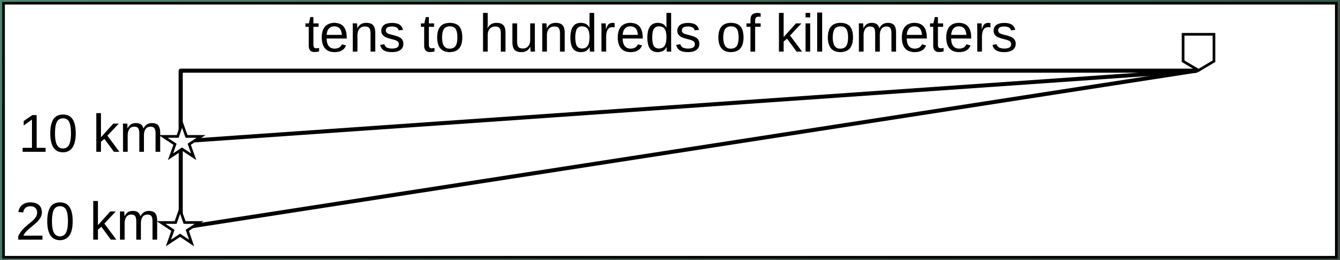


Depth estimation

ANSS station spacing ~280 km



- The distance between the station and the event is likely to be many kilometers. Therefore a small variation in focal depth (e.g. 5 km) will have little effect on the distance between the event and the station.
- Therefore the S-P time and P arrival time are insensitive to focal depth



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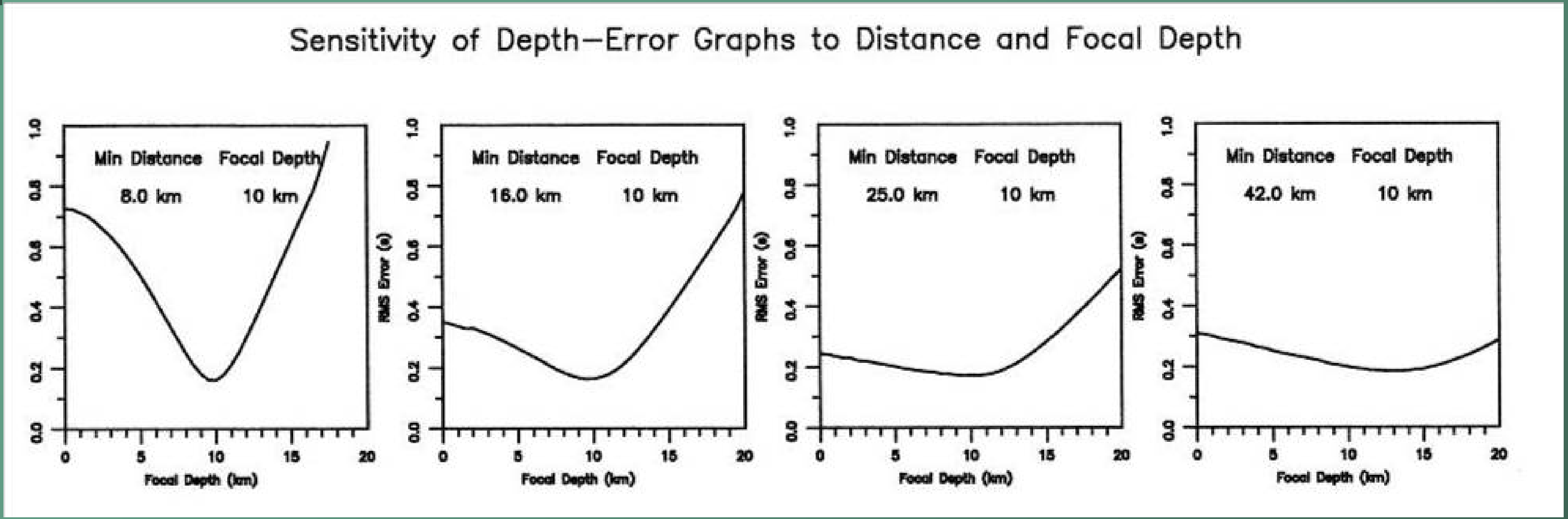

courtesy of Robert Mereu

- **Synthetic tests of variation in depth resolution - used in designing the network.**
- **As the distance for the quake to the nearest station increases the network becomes insensitive to the depth of the event (which was 10km for this test data).**

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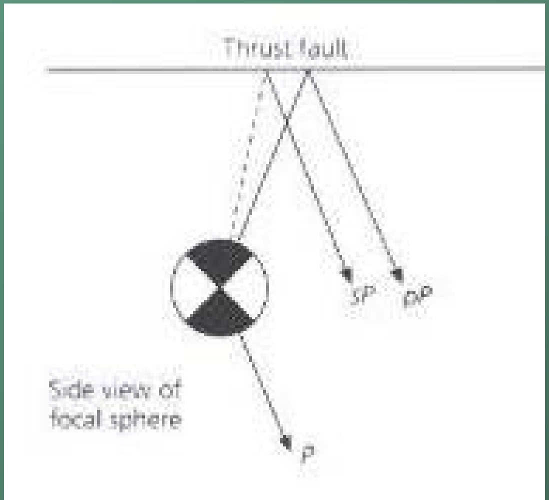
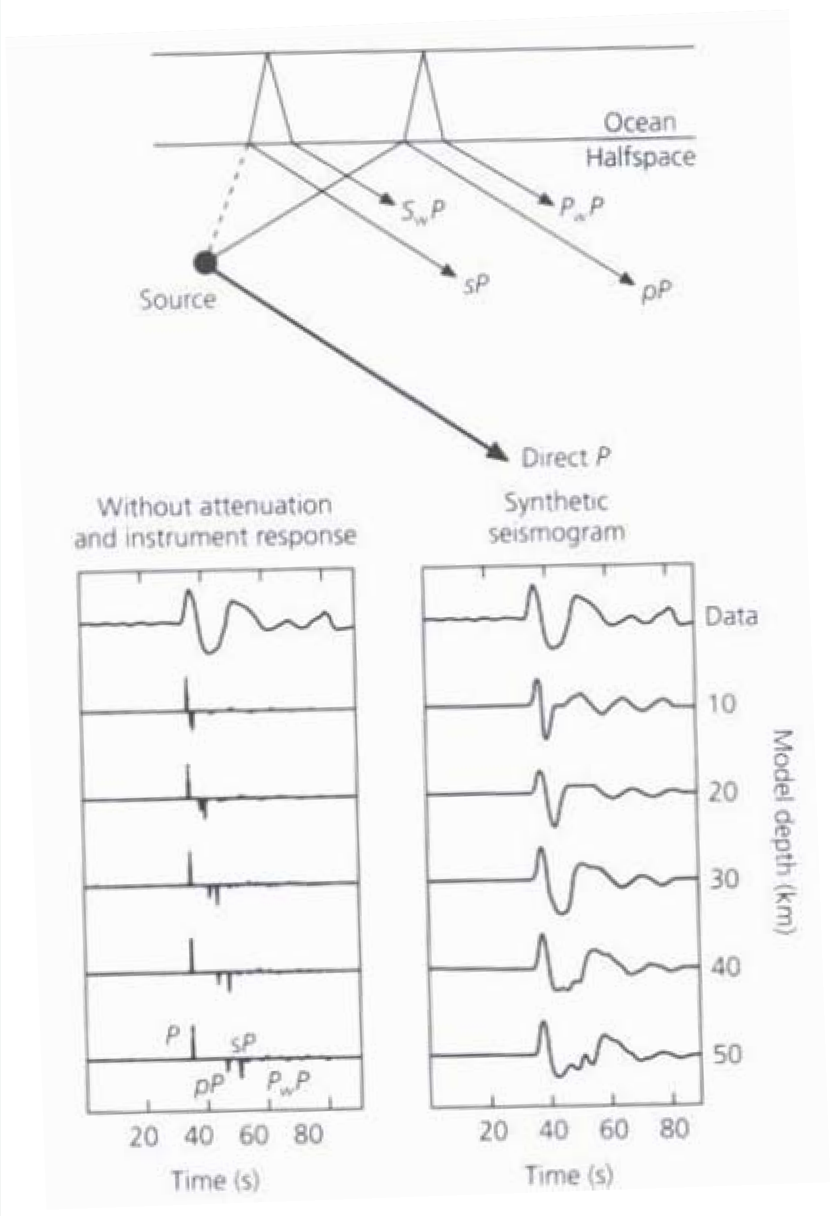
Depth – pP and sP

- The phases that reflect from the Earth surface near the course (pP and sP) can be used to get a more accurate depth estimate





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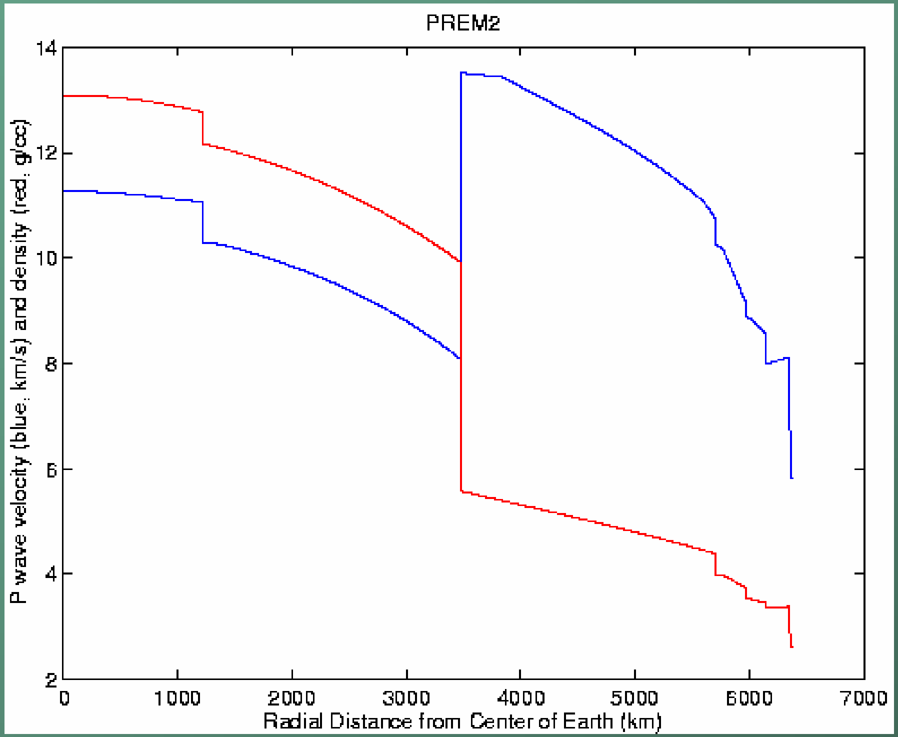
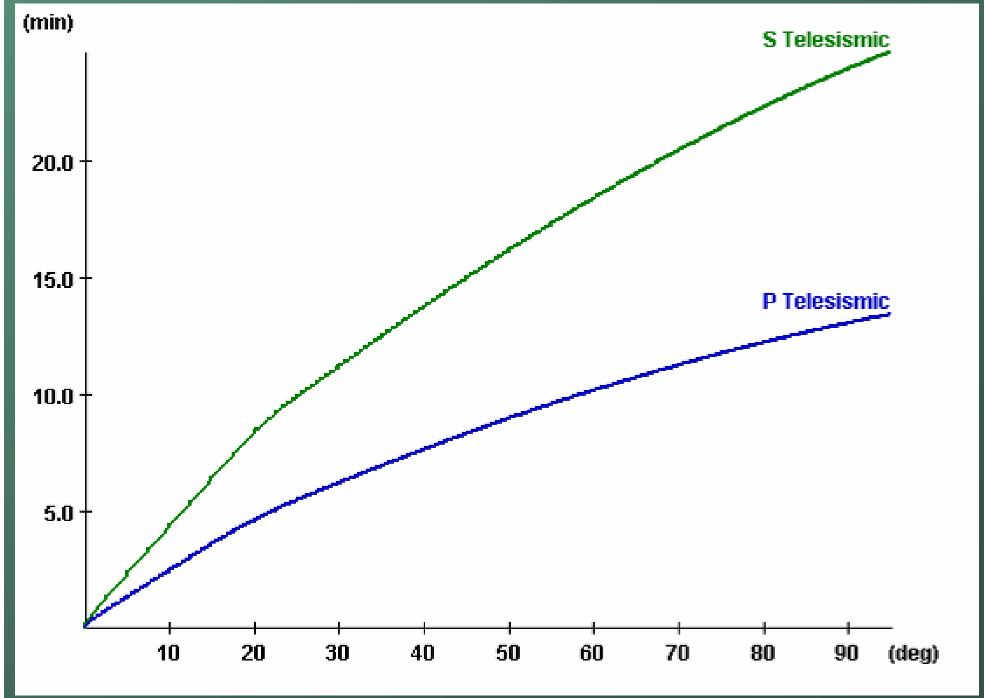
Stein and Wysession "An Introduction to Seismology, Earthquakes, and Earth Structure"

Velocity models

- For distant events may use a 1-D reference model (e.g. PREM) and station corrections





PREM model, Dziewonski & Anderson, 1981

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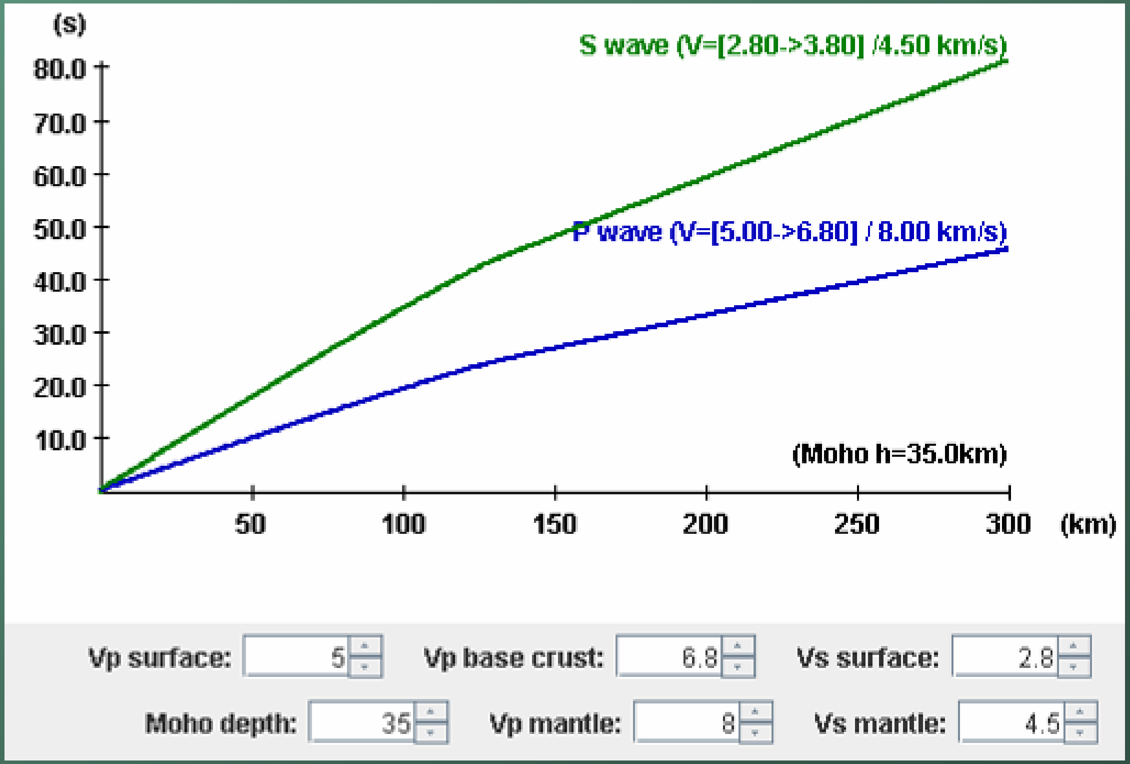
Local velocity model

- For local earthquakes need a model that represents the (1D) structure of the local crust.



SeisGram2K

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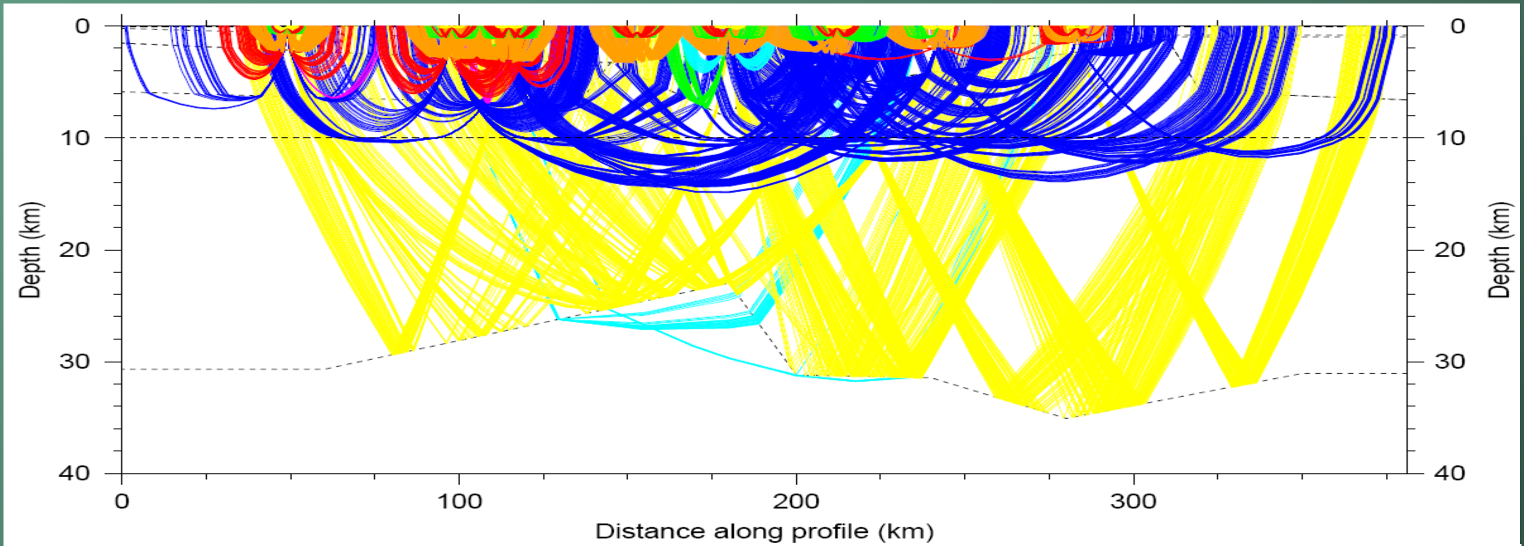
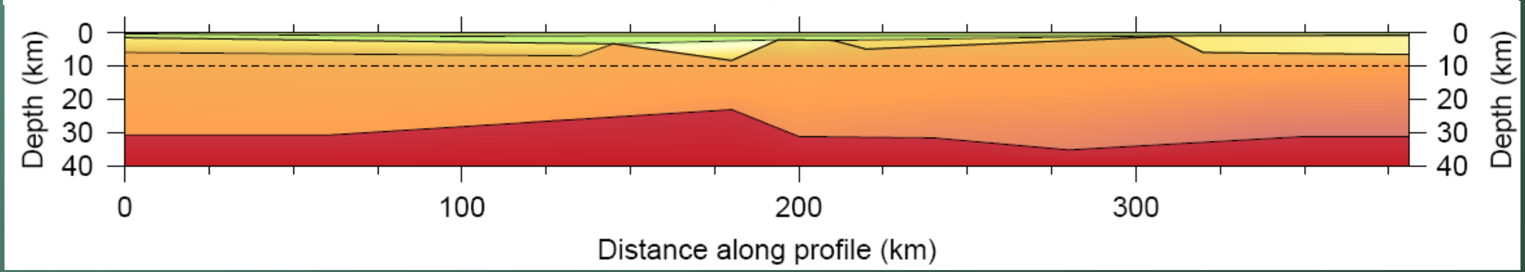
Determining the local velocity model

- **Refraction data the best for Moho depth and velocity structure of the crust.**





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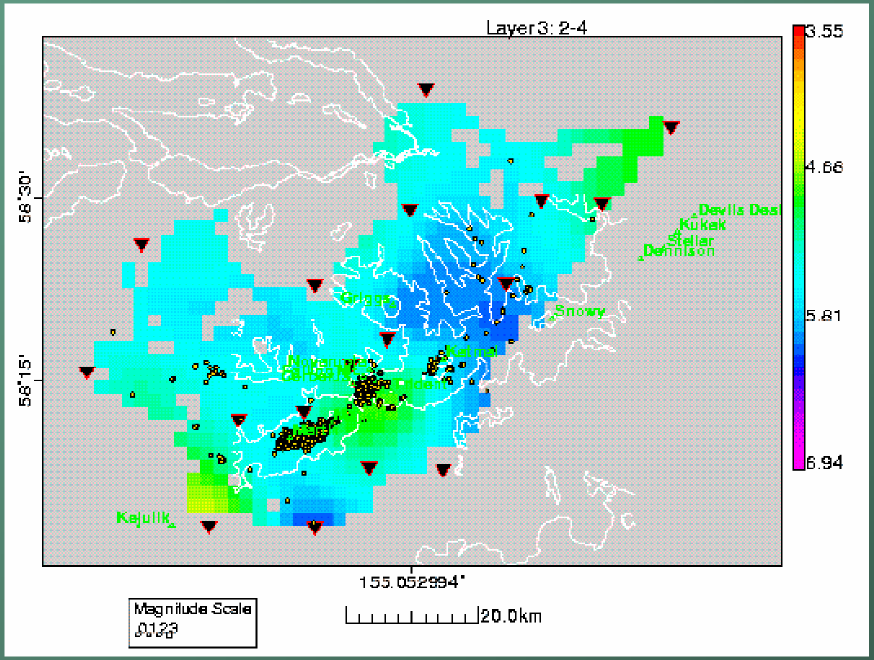
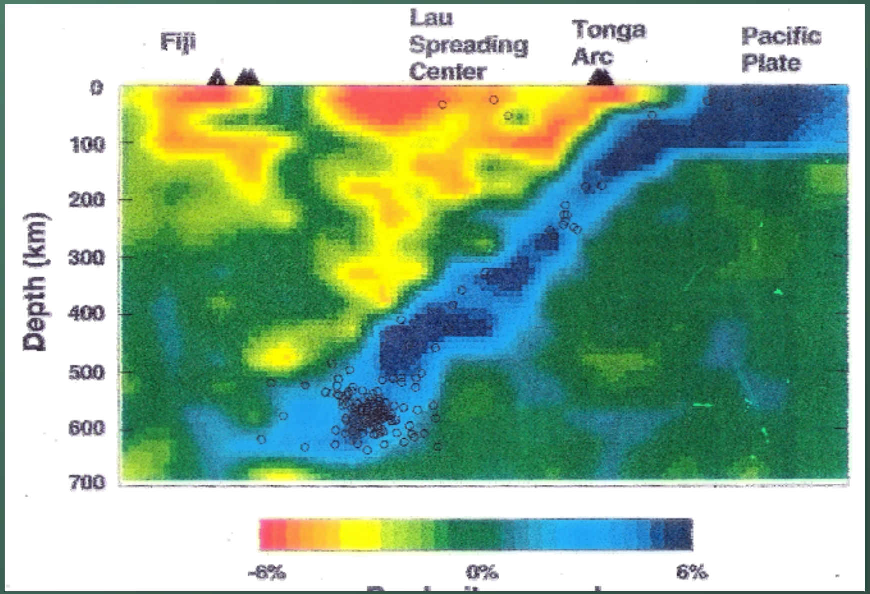
Tomography

- **Local tomography from local earthquakes can give crust and upper mantle velocities**
- **Regional/Global tomography from global events gives mantle velocity structure.**



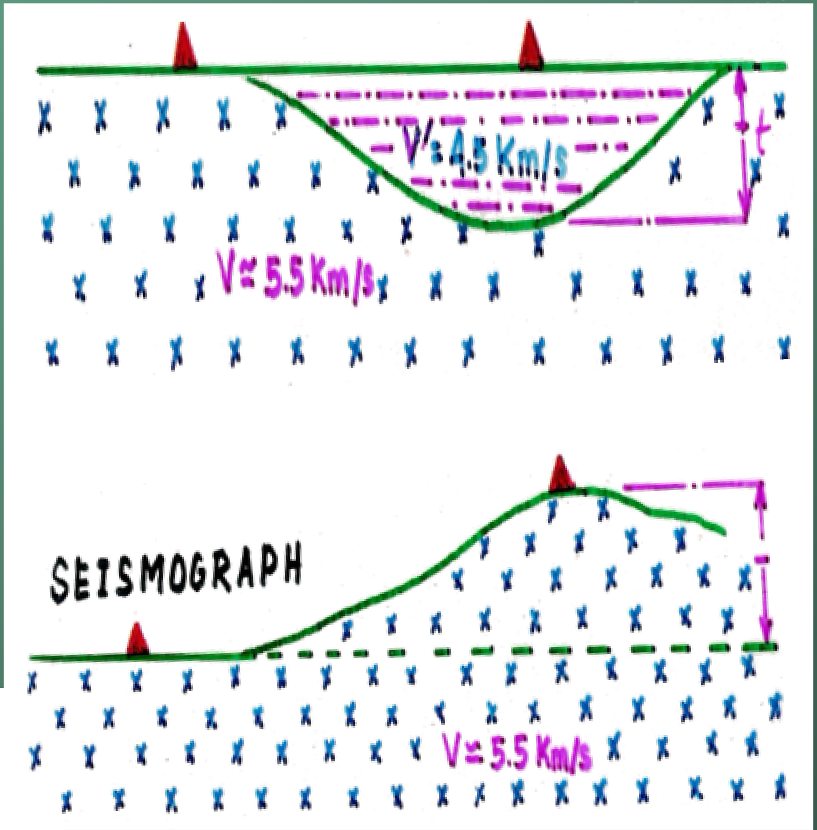


Seismic Tomography at the Tonga Arc Zone

(Zhao et al., 1994)

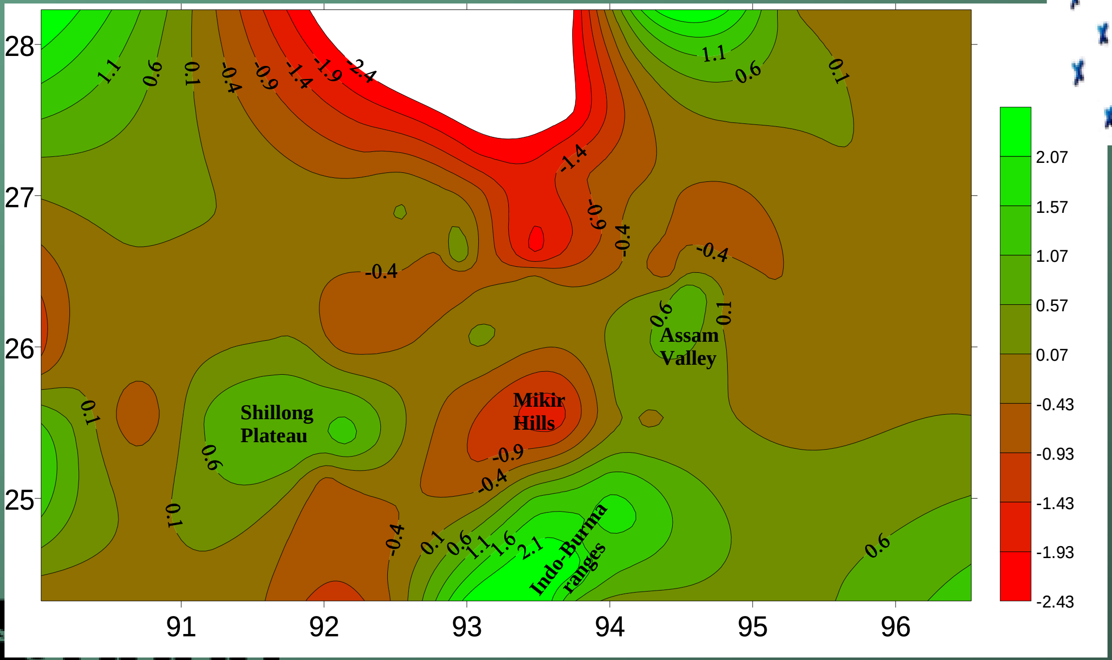
Station Corrections

- **Station corrections allow for local structure and differences from the 1D model**



Courtesy J R Kayal

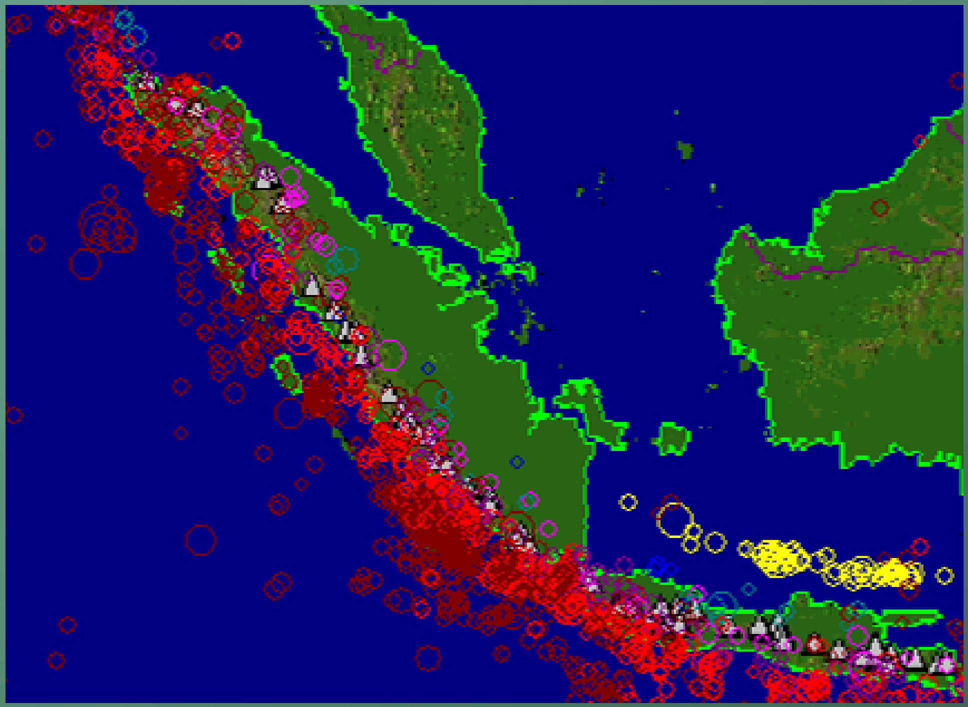
Contours of the P-wave Station Correction, NE India

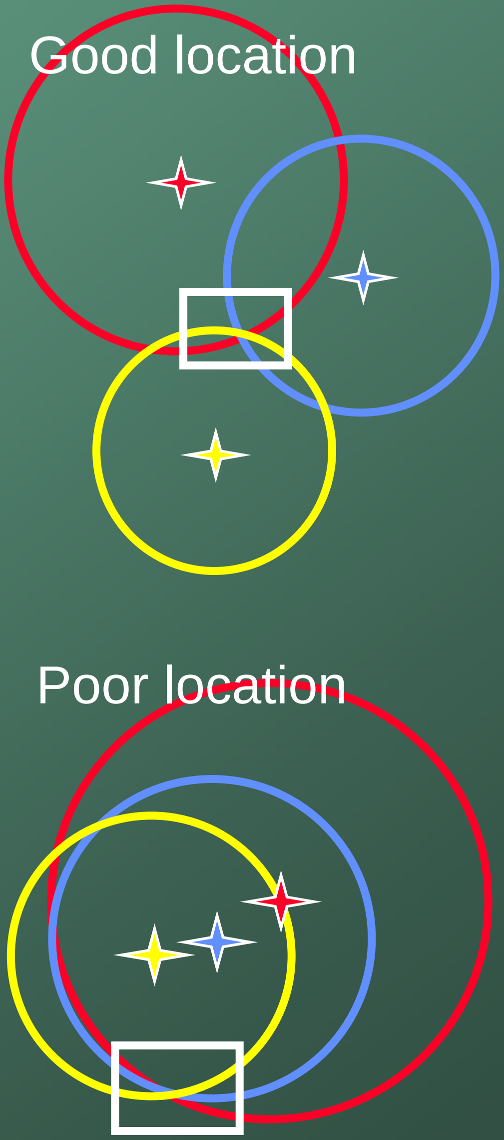


(Bhattacharya et al., 2005)

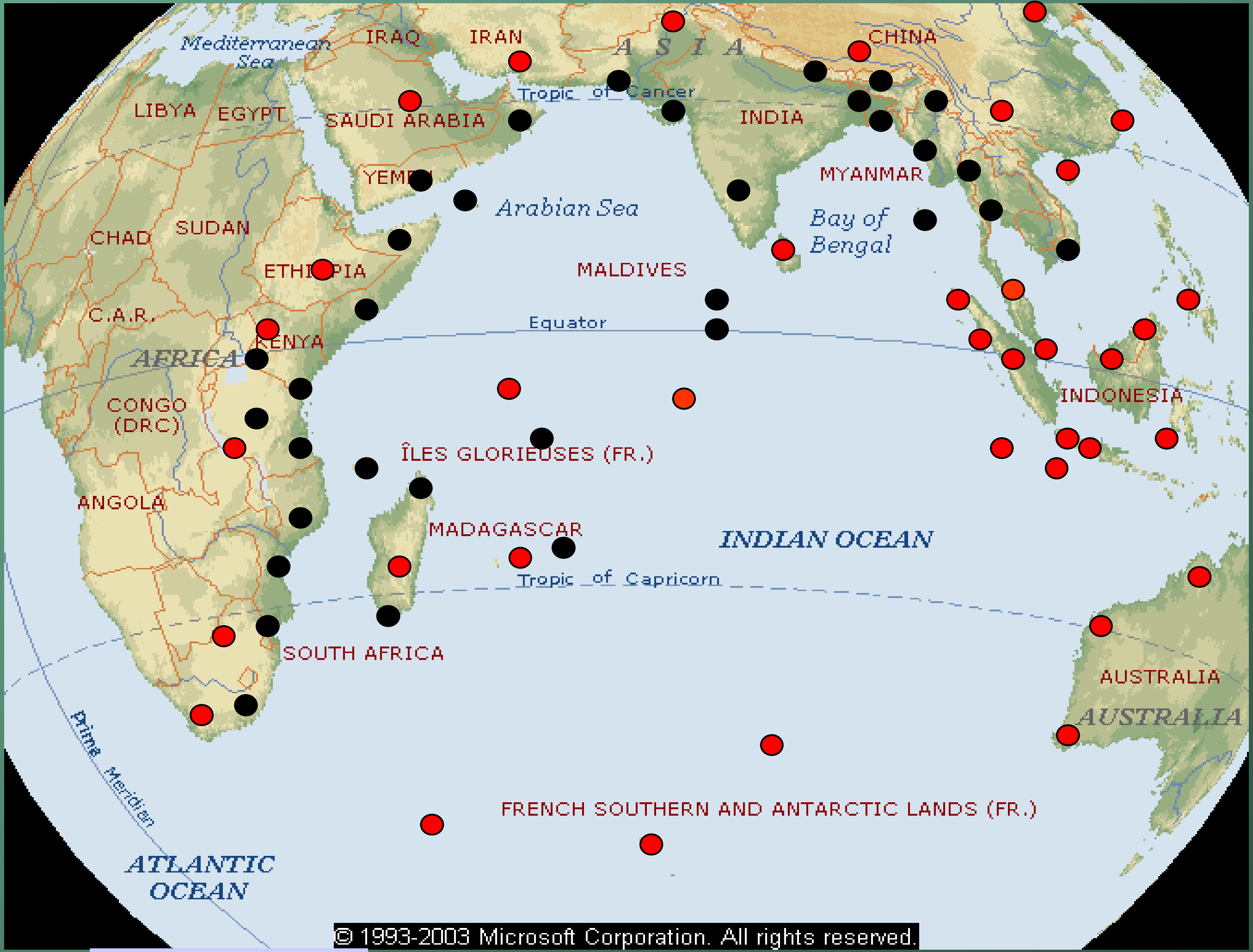
Location in subduction zones

- **Poor station distribution**





Stations in the Indian Ocean



Operational
Planned

Courtesy L. Kong

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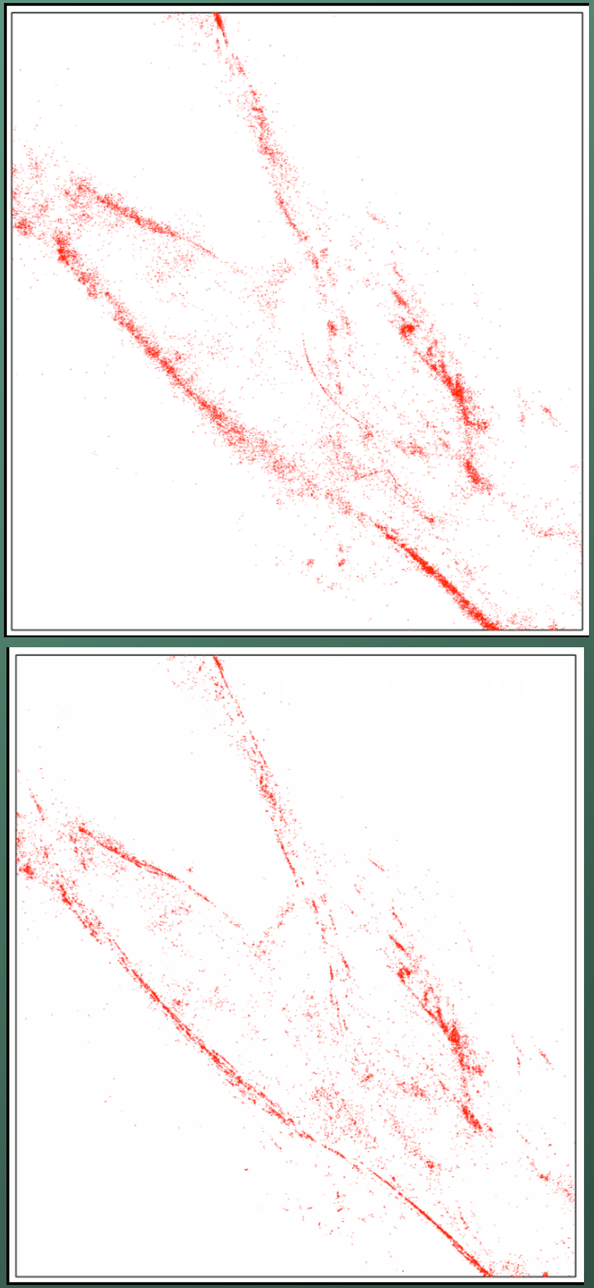
Relocation methods

- **Recalculate the locations using the relationship between the events.**
  - **Master Event Method**
  - **Joint hypocentral determination**
  - **Double difference method**



Waldhauser and Schaff "Improving Earthquake Locations in Northern California Using Waveform Based Differential Time Measurements"

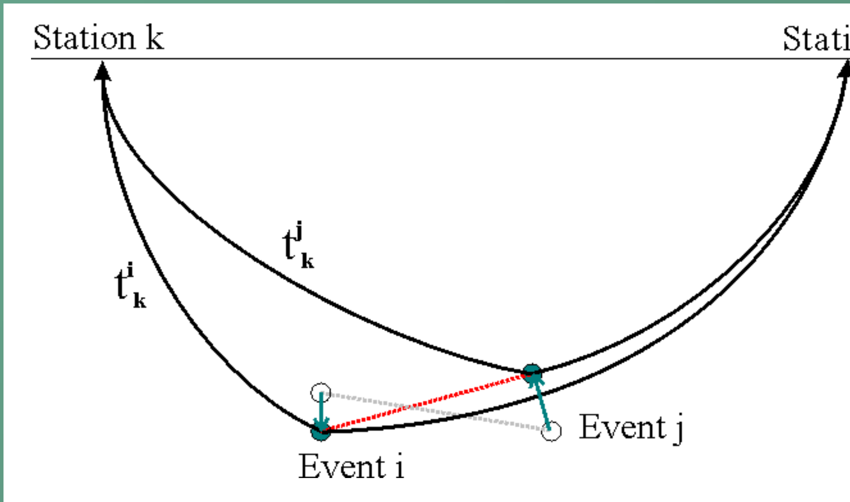
Master event relocation

- Select master event(s) – quakes with good locations, probably either the largest magnitude or event(s) that occurred after a temporary deployment of seismographs.
- Assign residuals from this event as the station corrections.
- Relocated other events using these station corrections.

Joint Hypocenter Determination (JHD)

- In JHD a number of events are located simultaneously solving for the station correction that minimizes the misfit for all events. (rather than picking one “master event” that is assumed to have good locations).

Double difference method



$$dr_k^{ij} = (t_k^i - t_k^j)^{obs} - (t_k^i - t_k^j)^{cal}$$

Double difference for event k – aim to minimize this residual

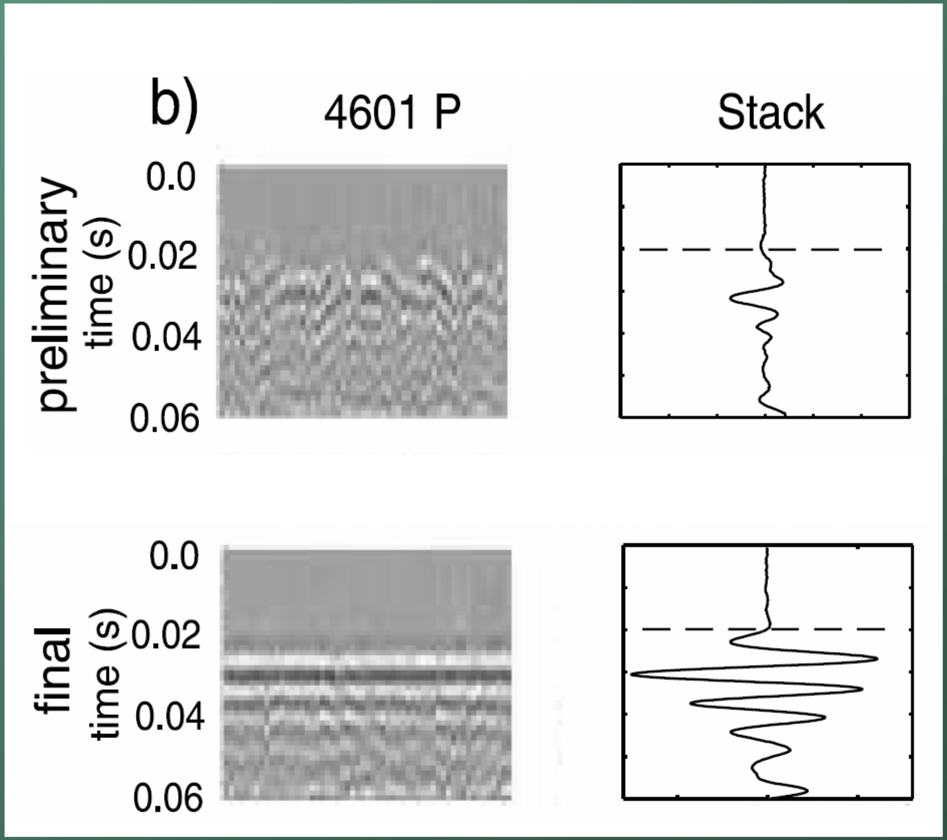
Difference in **observed** arrival time for stations i and j

Difference in **calculated** arrival time for stations i and j

- **This approach doesn't calculate station corrections.**
- **Instead the relative position of pairs of events is adjusted to minimize the difference between the observed and calculated travel time differences**

Cross-correlation to improve picks

- Phases from events with similar locations and focal mechanisms will have similar waveforms.
- realign traces to maximize the cross-correlation of the waveform.



Rowe et al 2002. Pure and Applied Geophysics 159

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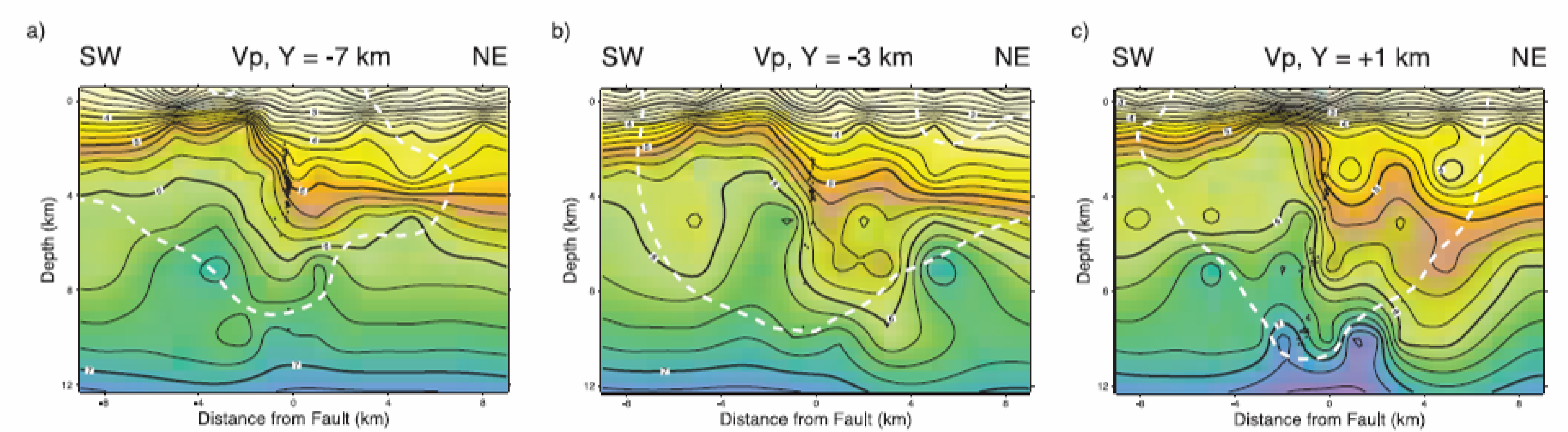
Simultaneous inversion

- Calculate the velocity structure and relocate the earthquakes at the same time.
- Needs very good coverage of ray paths through the model.

Model for Parkfield California – 15 stations, 6 explosions, 453 earthquakes



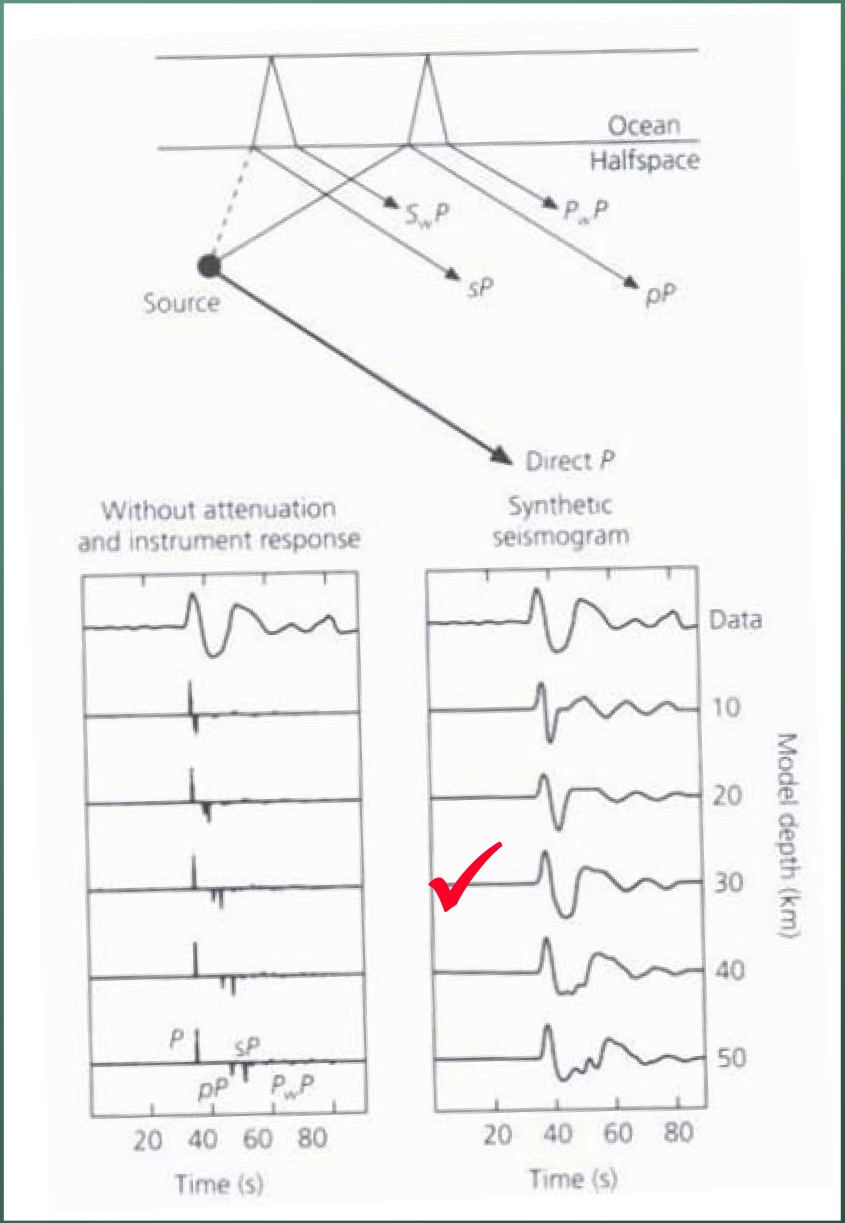
Thurber et al. 2003. Geophysical Research Letters

Some additional related topics...

- Waveform modeling
- Automated phase pickers
- location of great earthquakes

Waveform modeling

- Generate synthetic waveforms and compare to the recorded data to constrain the event



Stein and Wysession "An Introduction to Seismology, Earthquakes, and Earth Structure"

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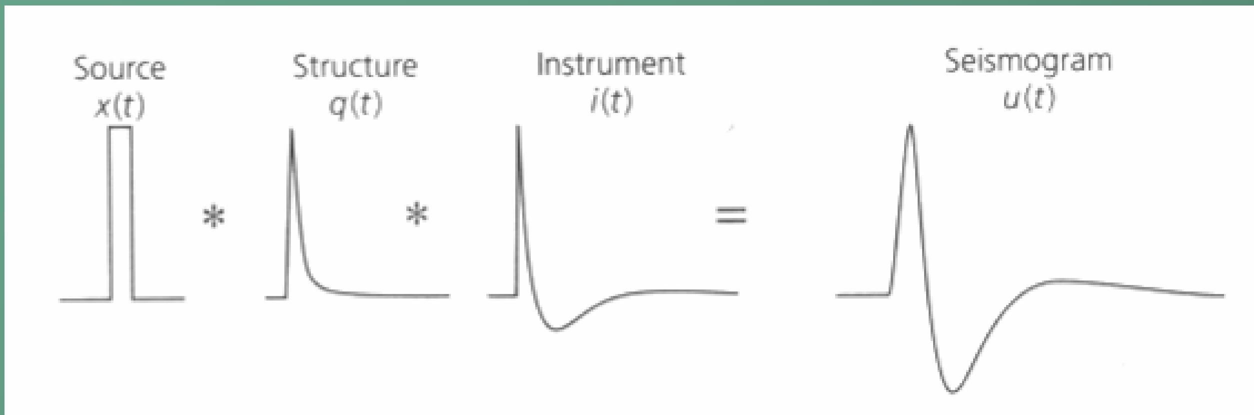
Waveform modeling



$$u(t) = x(t) * e(t) * q(t) * i(t)$$

$$U(\omega) = X(\omega)\, E(\omega)\, Q(\omega)\, I(\omega)$$

seismogram

source time function

reflections & conversions at interfaces

attenuation

instrument response

Construction of the synthetic seismogram

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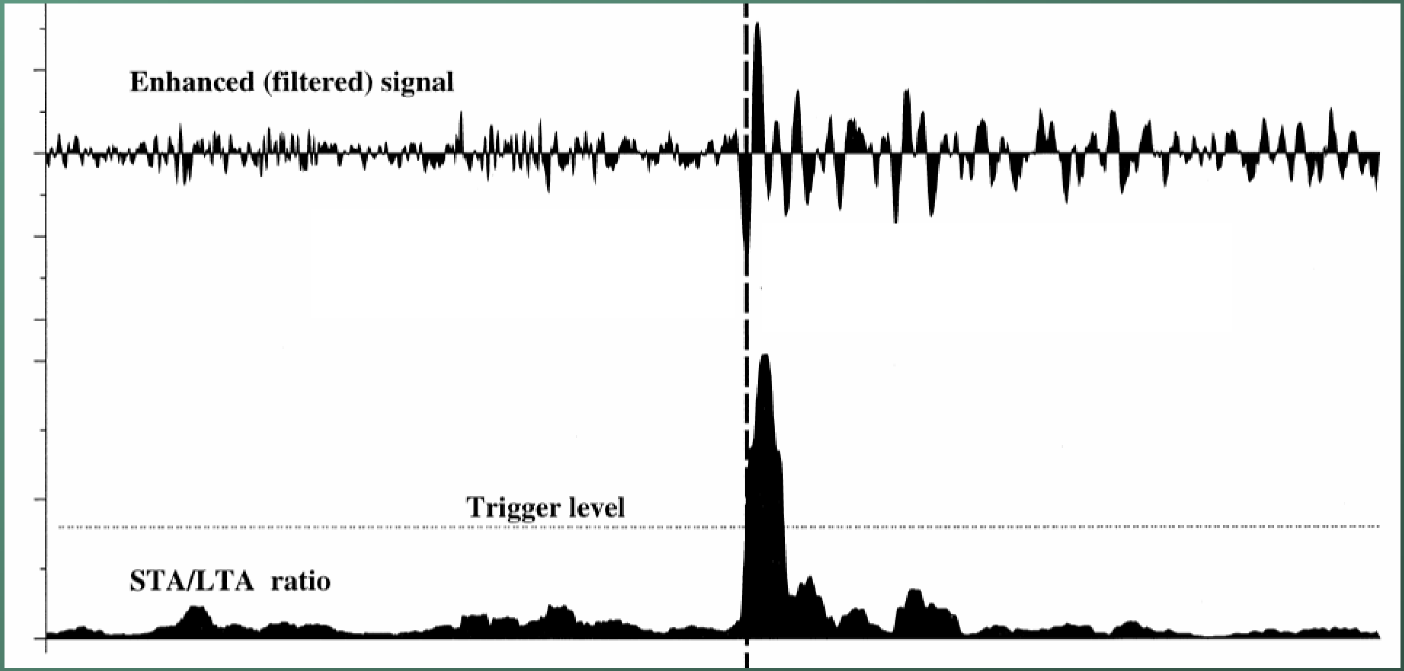
Automatic phase picks

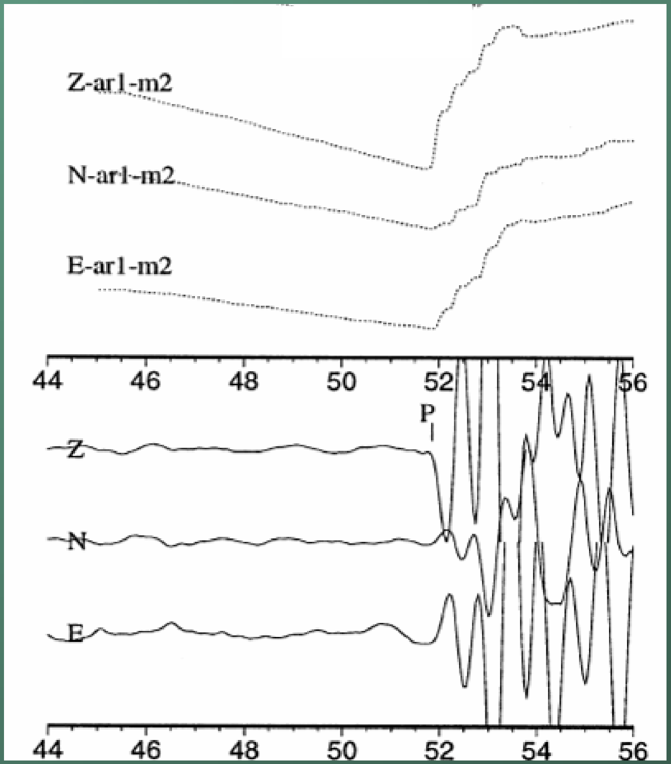
- Short term average - long term average (STA/LTA) – developed in the 1970s, still used by Earthworm and Sac2000



Sleeman and von Eck 1999, Physics of Earth and Planetary Interiors 113

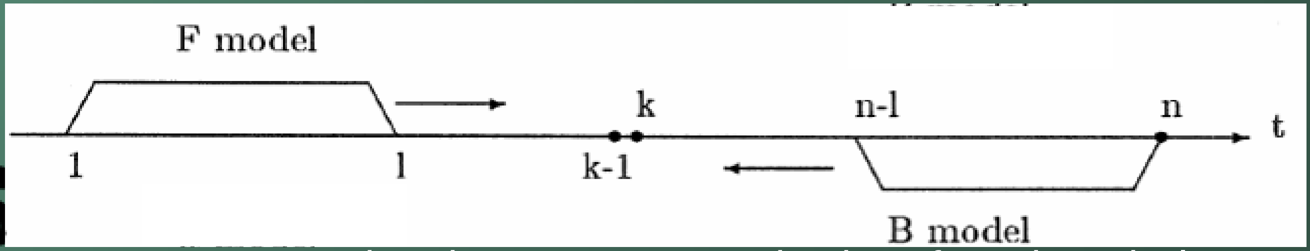
Autoregression analysis

- Autoregression (AR) models the seismogram as predictable signal + noise



$$u_t = \sum_{i=1}^{m} A_i u_{t-i} + v_t$$

- Find the point at which predictable signal can be identified using Akaike Information Criterion (AIC) from the AR of series in the noise and in the phase.



Leonard and Kennett 1999, Physics of Earth and Planetary Interiors 113

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CUSUM algorithm

- **Looks for a change in the cumulative sum of a statistic that defines a change in properties.**
- **Calculate a CUSUM of a statistic and subtract the trend (converts changes in the trend to minima)**

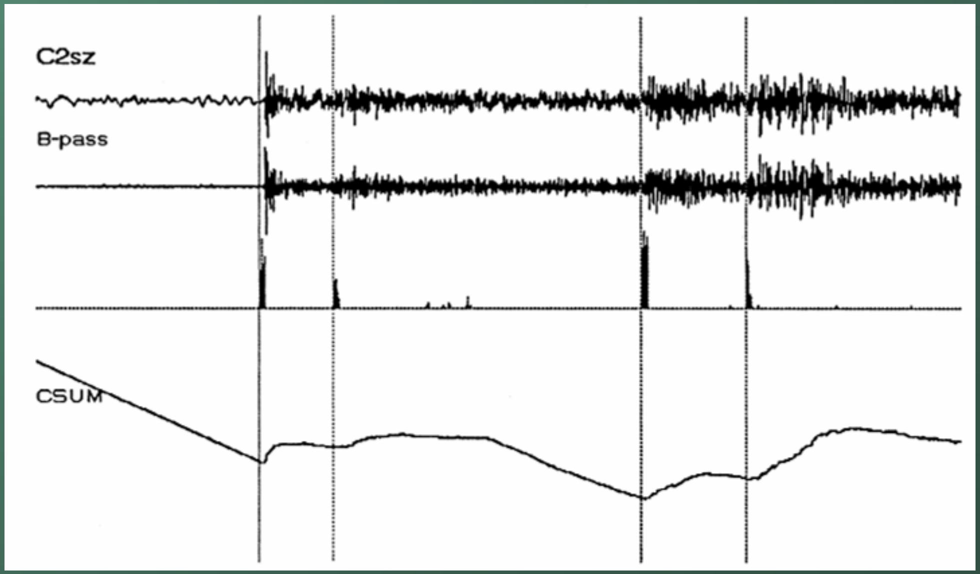
look for minima in this function

$$D_{\kappa} = \frac{C_k}{C_T} - \frac{\kappa}{T}$$

Where C_k is the cumulative squared amplitude (up to point K)

$$C_k = \sum_{i=1}^{k} x_i^2$$

and C_T is the sum of x^2 over the whole window of T points)



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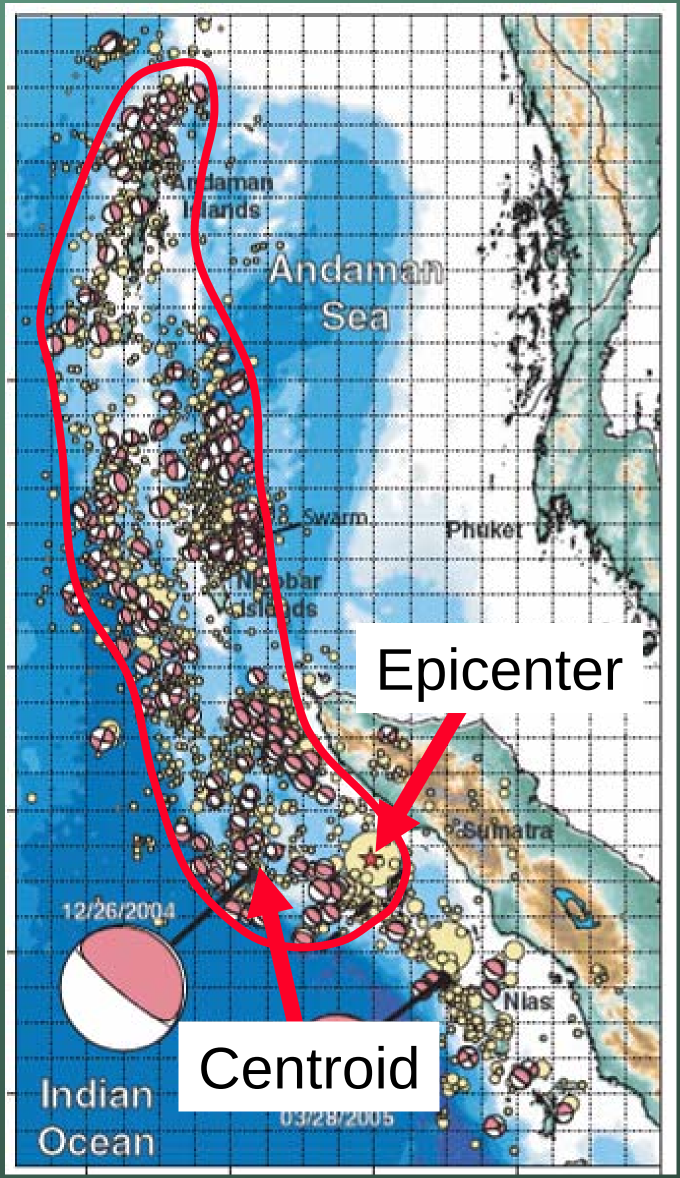
Der and Shumway 1999, Physics of Earth and Planetary Interiors 113

Location of Great Earthquakes

- With great earthquakes the slip area is very large (hundreds of kilometers)
- For hazard assessment the epicenter and centroid are not very informative. Need to rupture area, but this is not available in time for tsunami warnings/disaster management.



Lay et al 2006, Science 308