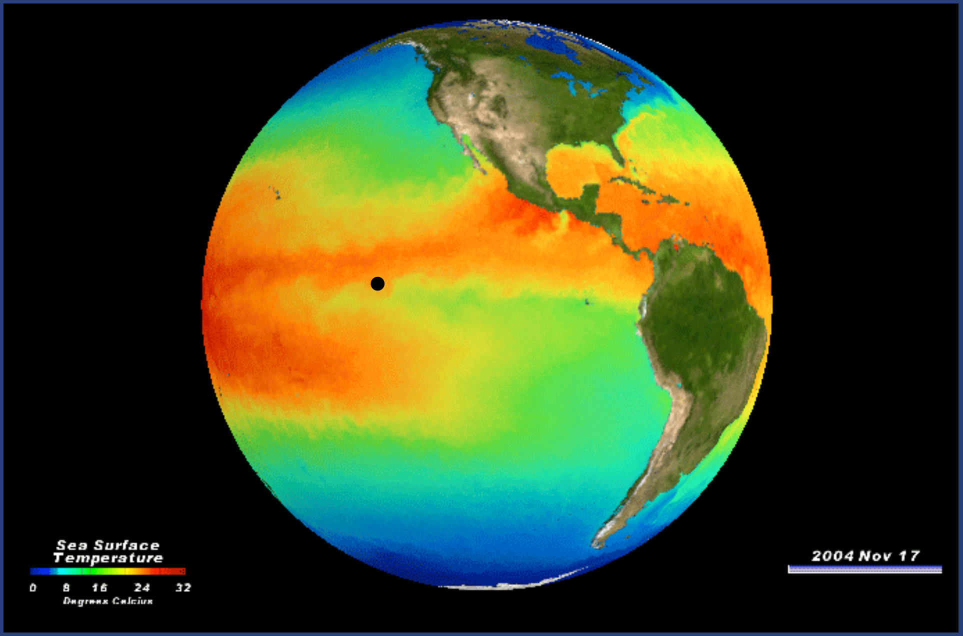
A Generalized Ekman Model for Frontal Regions



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William S. Kessler
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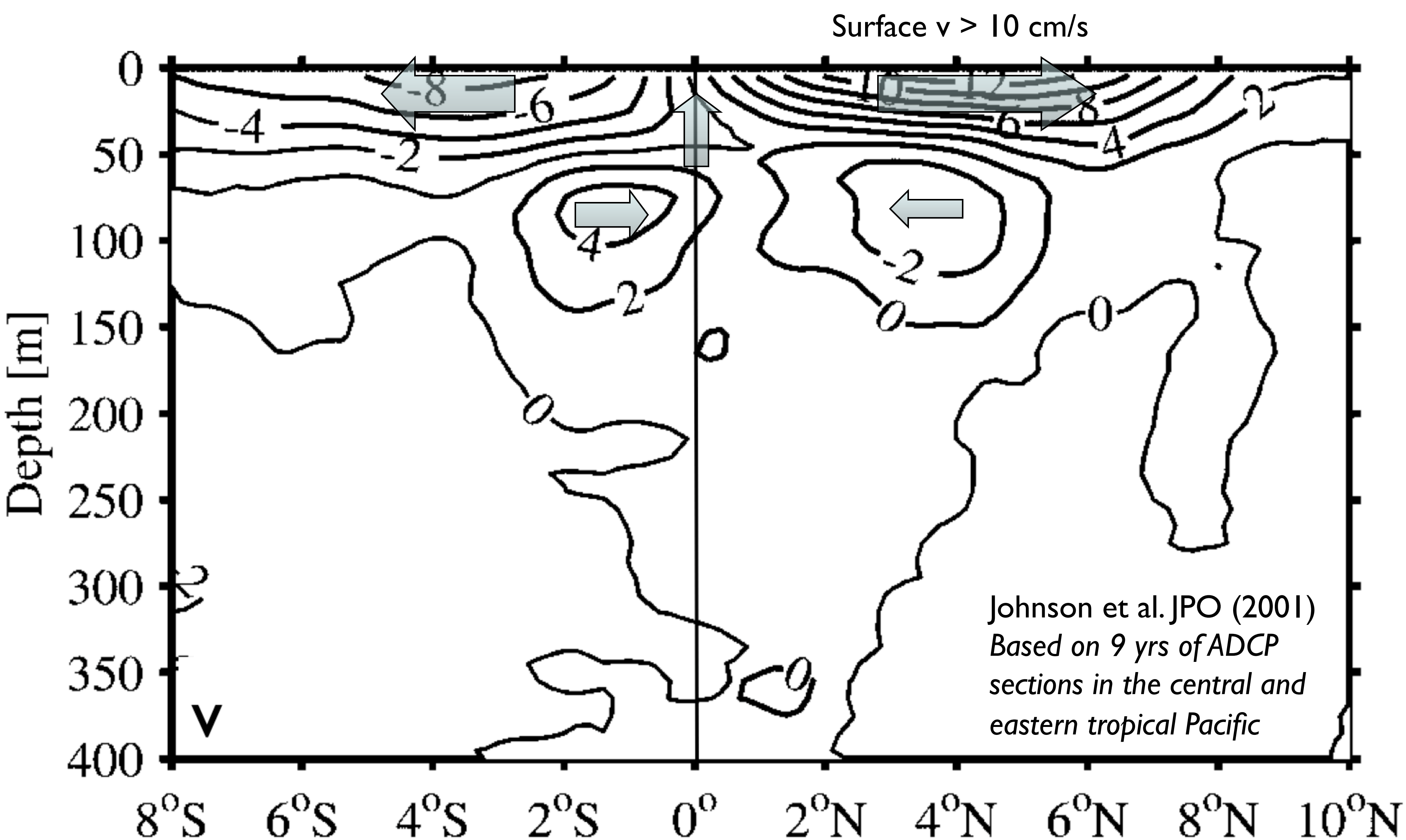
UW Oceanography, affiliate Professors

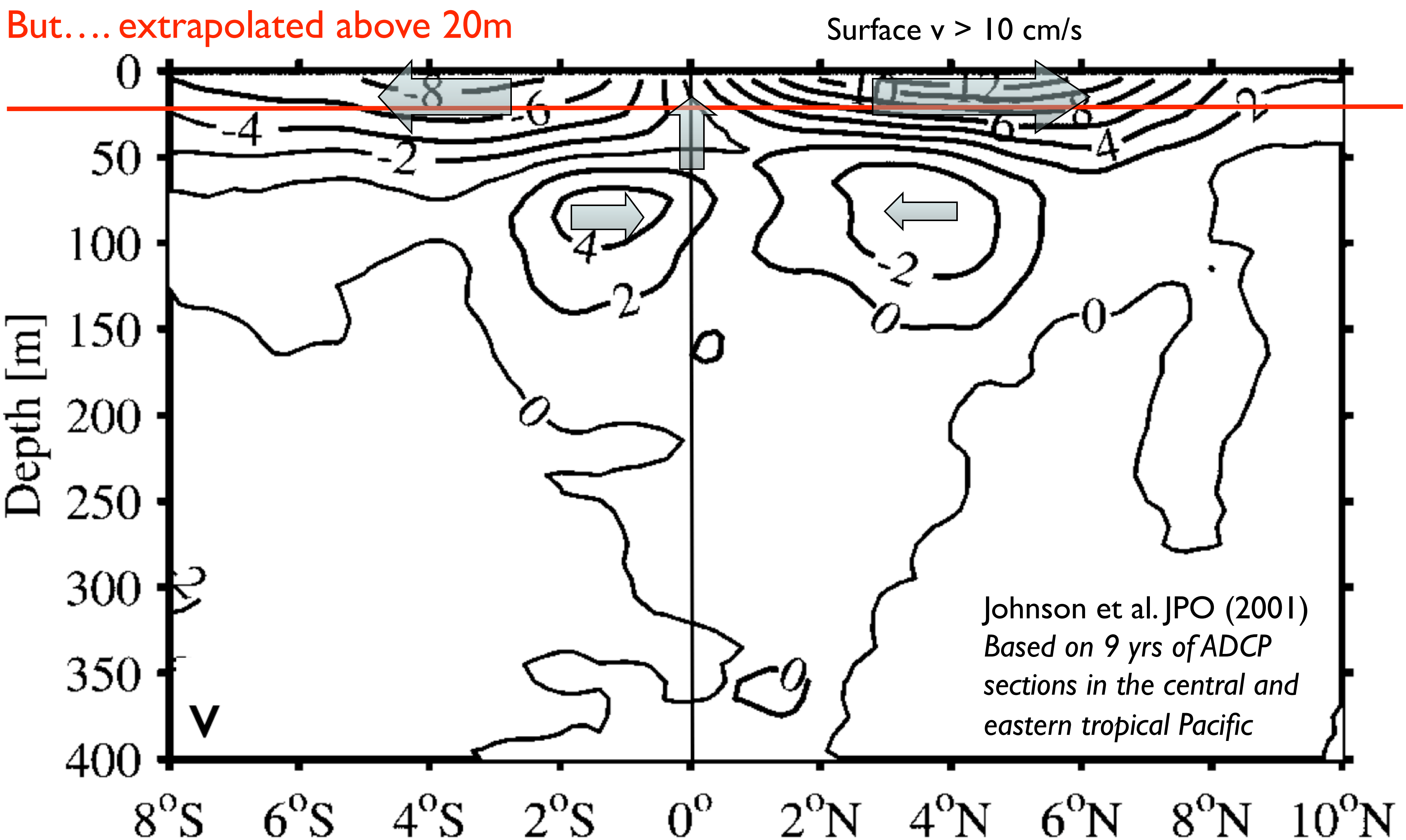
Cronin, M. F. and W. S. Kessler, 2009: Near-surface shear flow in the tropical Pacific cold tongue front. *J. Phys. Oceanogr.*, 39, 1200-1215.

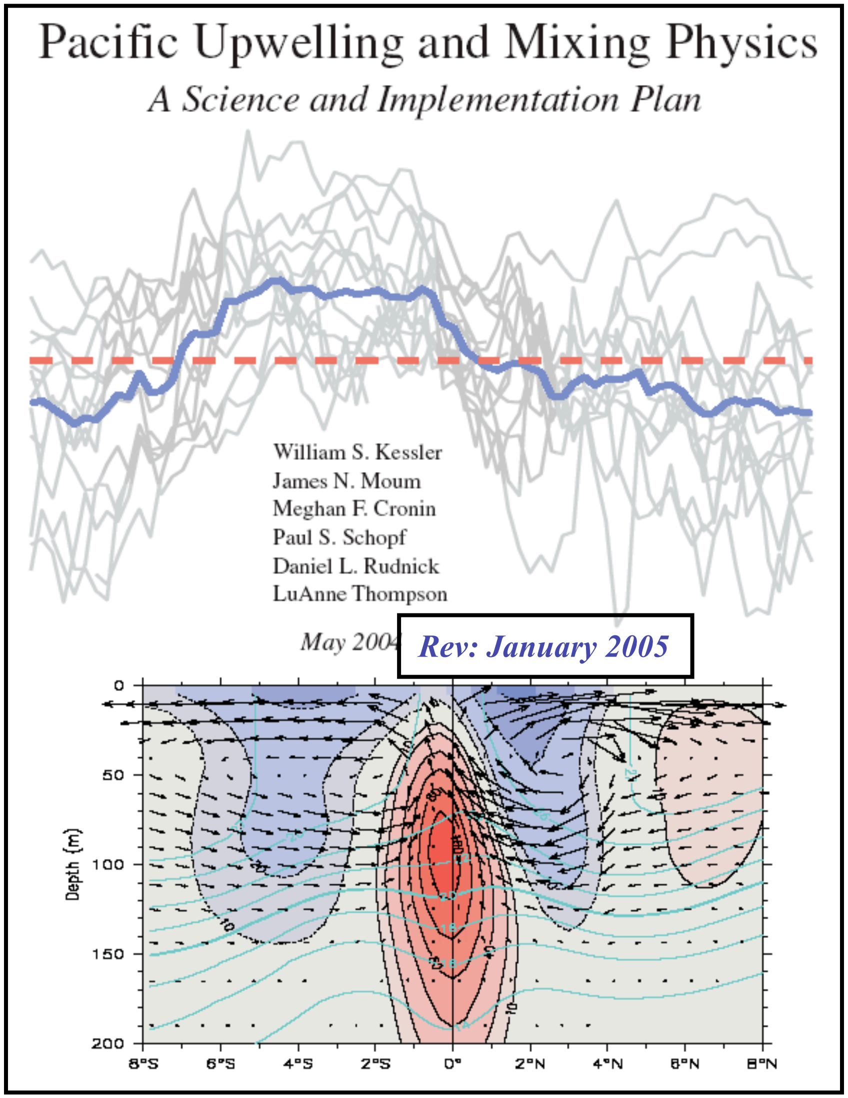
The “classical” picture of equatorial circulation

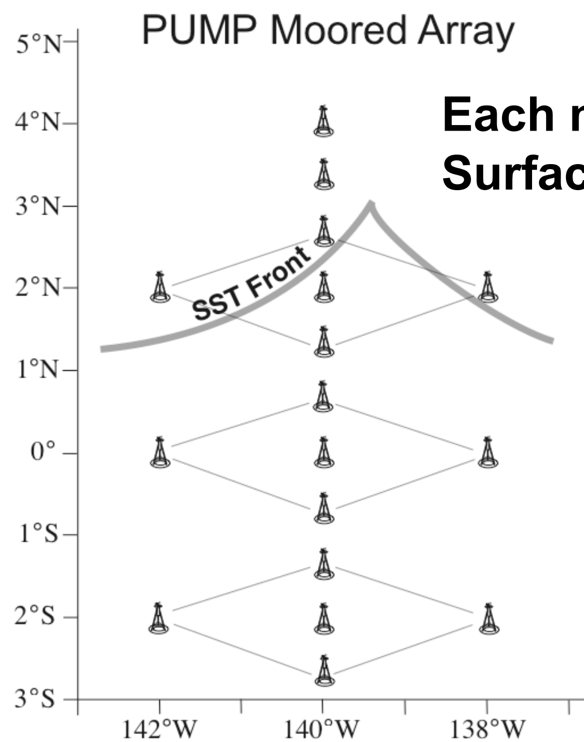


Johnson et al. JPO (2001)
Based on 9 yrs of ADCP sections in the central and eastern tropical Pacific

Neither shipboard, nor moored upward-looking ADCPs measure currents above 20 m. *Is there shear above 25m?*

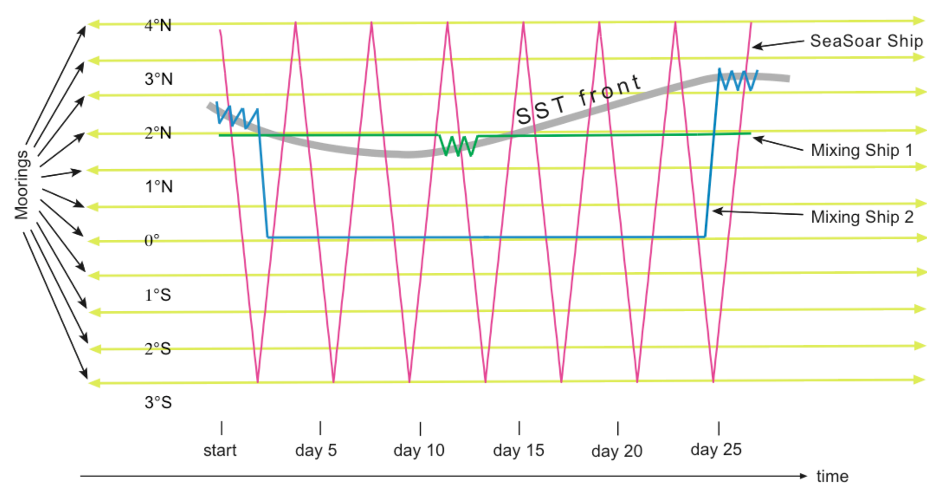






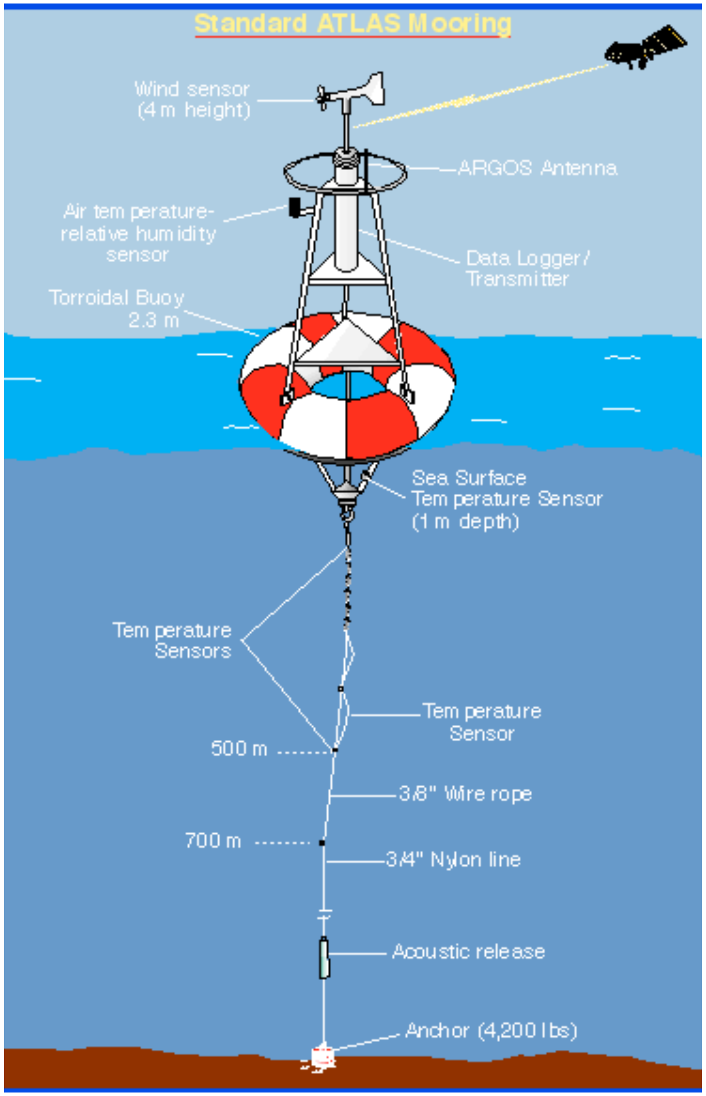
Each mooring is a Surface buoy+ADCP

PUMP Intensive Observing Periods



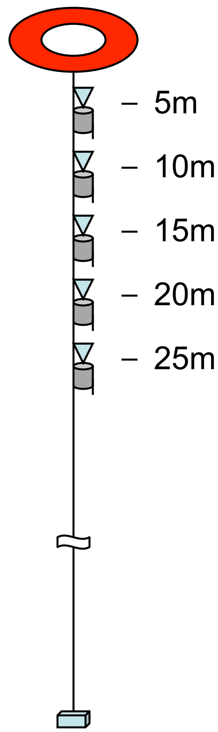
PUMP ~~will~~ would have put mixing observations in context of 3-d circulation.

9 months of near-surface velocity & temperature at 2°N, 140°W



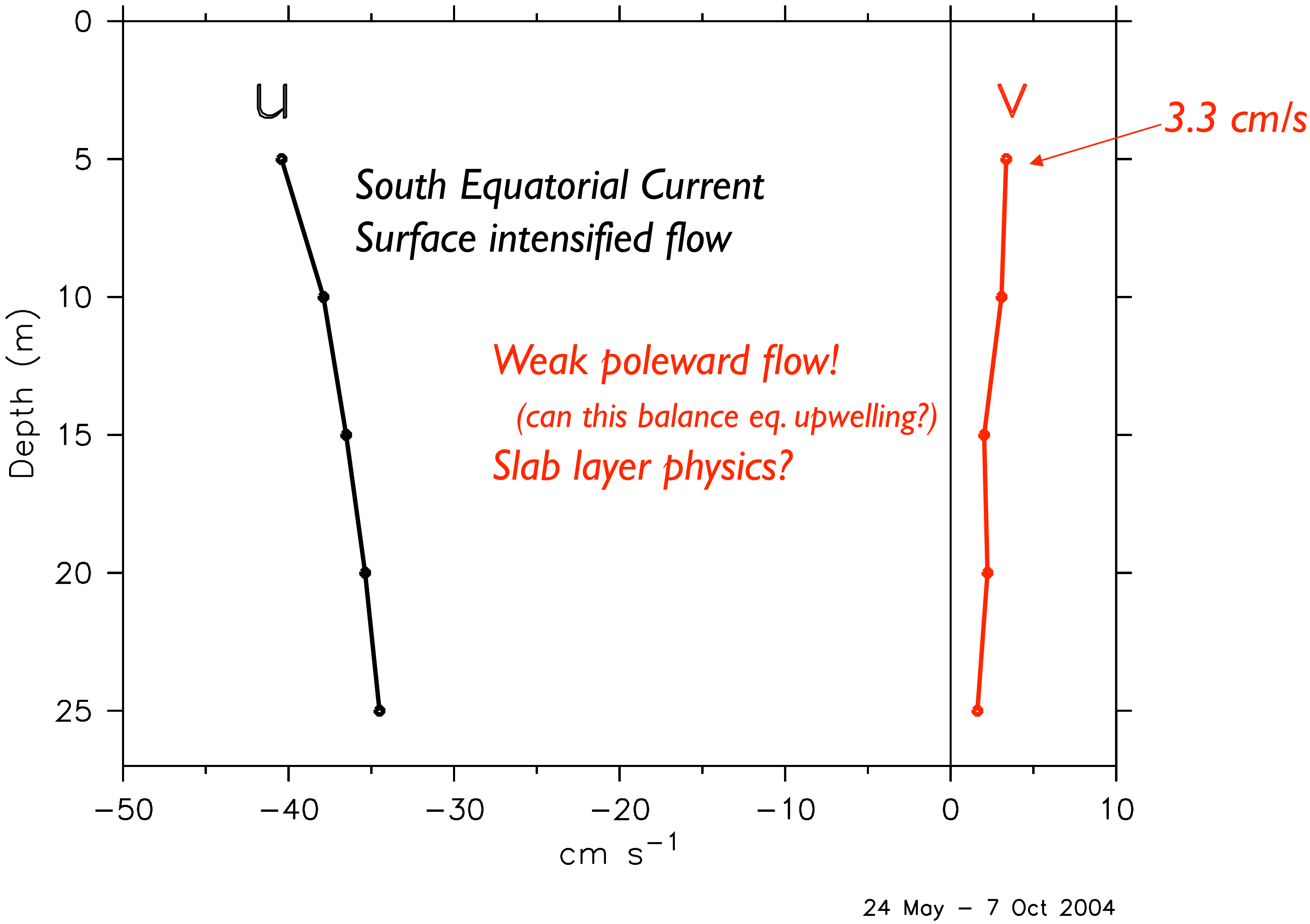
	2003	2004	2005
SONTEK sensors	TAO/EPIC 95W	2N140W Test Mooring	KEO & IO

| May 04 – Feb 05 |

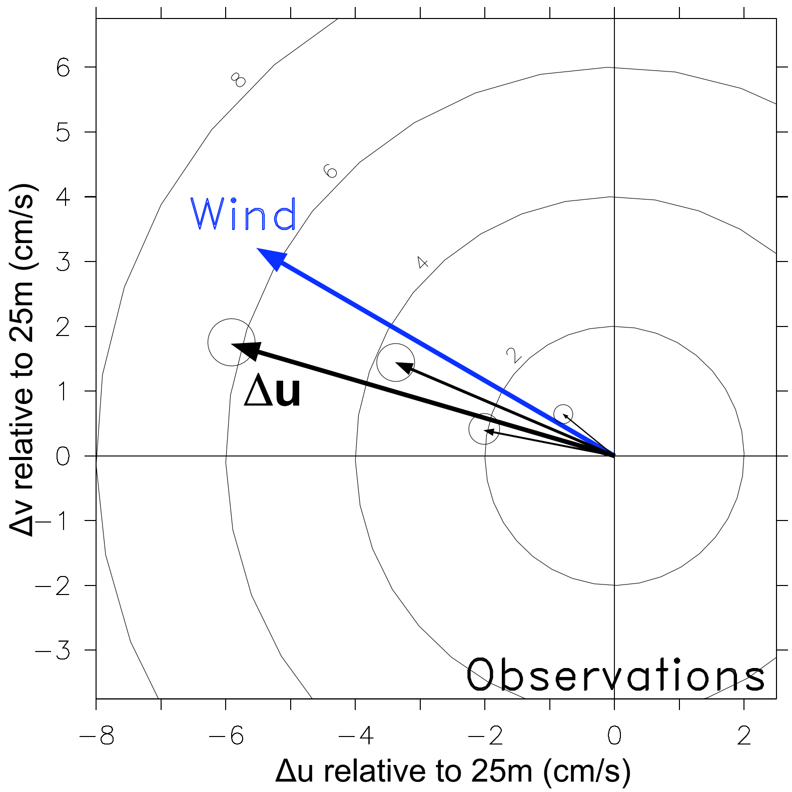


- *5 Sonteks (acoustic Doppler current meters) were placed on a test mooring near the 2ºN,140ºW TAO mooring.*
- *Each Sontek had a thermistor.*

Mean near-surface currents at 2°N, 140°W



Observed wind and currents



Mean for 24-May-2004 to 7-Oct-2004

Vector Hodographs

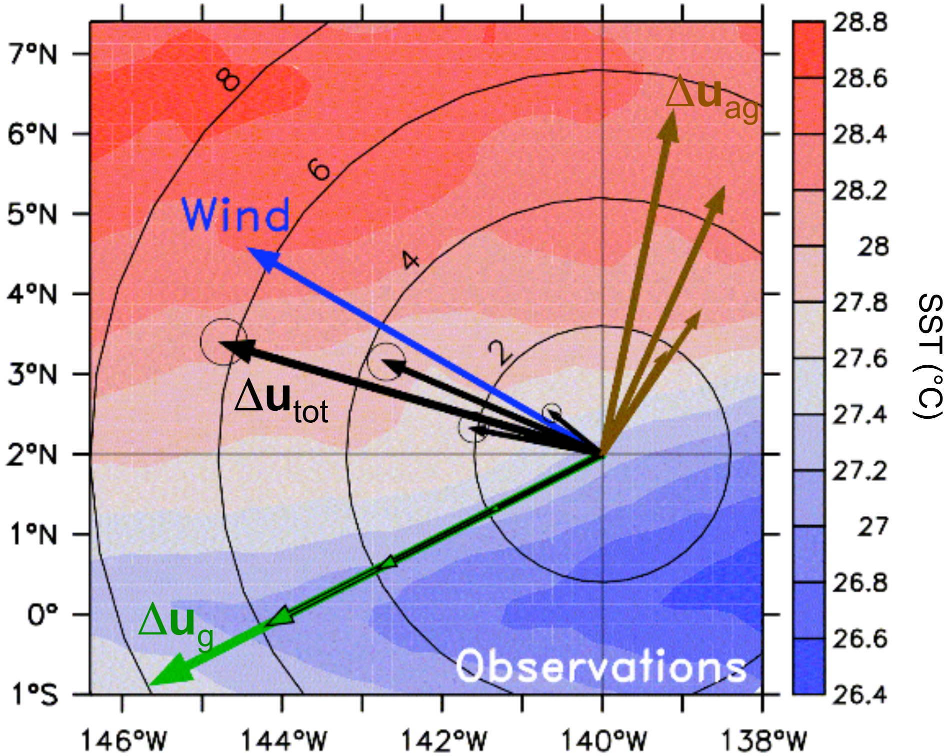
magnitude scale circles have units:

Blue: wind speed (m/s)

Black: current shear, **Δu** *(cm/s)*

$$\Delta \mathbf{u} = \mathbf{u} - \mathbf{u}(z=25\text{m})$$

Observed wind, currents and SST



(Ekman depth D_{ek}= 25 m ?)

Observed ageostrophic currents relative to 25 m has Ekman-like spiral 70° to right of wind. But…

Vector Hodographs

magnitude scale circles have units:

Blue: wind speed (m/s)

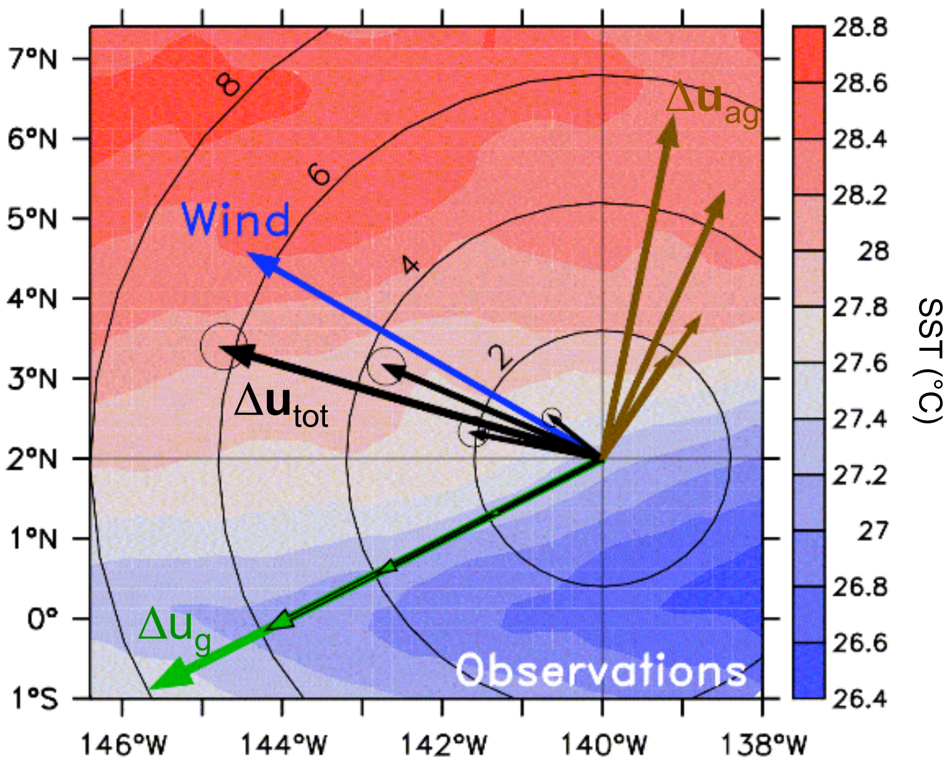
Black: current shear, $\Delta \mathbf{u}$ *(cm/s)*

$\Delta \mathbf{u} = \mathbf{u} - \mathbf{u}(z=25\text{m})$

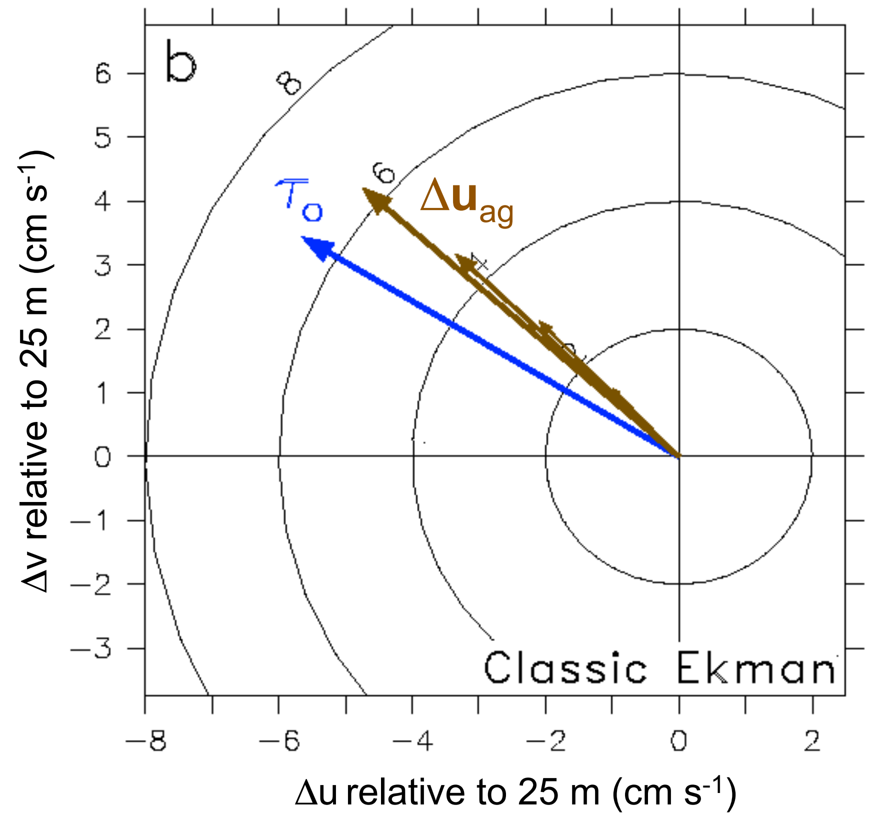
Green: geostrophic shear, $\Delta \mathbf{u}_g$
(estimated from observed SST)

Brown: ageostrophic shear, $\Delta \mathbf{u}_{ag}$

$\Delta \mathbf{u}_{ag} = \Delta \mathbf{u}_{tot} - \Delta \mathbf{u}_g$



(Ekman depth D_{ek}= 25 m ?)



(Ekman depth D_{ek}= 80 m, with $\nu \sim 1.6 \times 10^{-2}$ m^2/s from observed shear & τ_0)

Observed ageostrophic currents relative to 25 m has Ekman-like spiral 70° to right of wind. But…

Classic Ekman spiral has Δu_{ag} shear aligned slightly to the right of the wind stress.

"**Classic Ekman Model**" (Ekman 1905):

Assume steady, linear flow; uniform density and viscosity; driven by surface wind stress, no drag at $z = -H \sim -\infty$. Solve for $\boldsymbol{u_a}(z)$.

Equation of motion:

$$ifu = -\frac{1}{\rho}\nabla P + \nu\frac{\partial^2 u}{\partial z^2}$$

Boundary conditions:

$$at\ \ z = 0: \quad \frac{\partial u}{\partial z} = \frac{\boldsymbol{\tau}_0}{\rho\nu}$$

$$at\ \ z = -H: \quad u = 0$$

(Since density is uniform, No geostrophic shear)

$$note: \quad u = u_g + u_a,\ \ where: \quad u_g = \frac{i}{\rho f}\nabla P \ \ and \ \ \frac{\partial u_g}{\partial z} = \frac{ig\alpha}{\rho f}\nabla T = 0$$

"**Classic Ekman Model**" (Ekman 1905):

Assume steady, linear flow; uniform density and viscosity;
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The familiar Classic Ekman equations:

Equation of motion:

$$ifu = -\frac{1}{\rho}\nabla P + \nu \frac{\partial^2 u}{\partial z^2} \quad \rightarrow \quad ifu_a = \nu \frac{\partial^2 u_a}{\partial z^2}$$

Boundary conditions:

$$at\ \ z = 0: \quad \frac{\partial u}{\partial z} = \frac{\tau_0}{\rho \nu} \quad \rightarrow \quad at\ \ z = 0: \quad \frac{\partial u_a}{\partial z} = \frac{\tau_0}{\rho \nu}$$

$$at\ \ z = -H: \quad u = 0 \quad \rightarrow \quad at\ \ z = -H: \quad u_a = 0$$

$$note: \quad u = u_g + u_a, \ \ where: \quad u_g = \frac{i}{\rho f}\nabla P \ \ and \ \ \frac{\partial u_g}{\partial z} = \frac{ig\alpha}{\rho f}\nabla T = 0$$

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Assume steady, linear flow; uniform density and viscosity; driven by surface wind stress, no drag at $z = -H \sim -\infty$. Solve for $\boldsymbol{u_a}(z)$.

$$D_{ek} = \sqrt{\frac{2\nu}{f}}$$

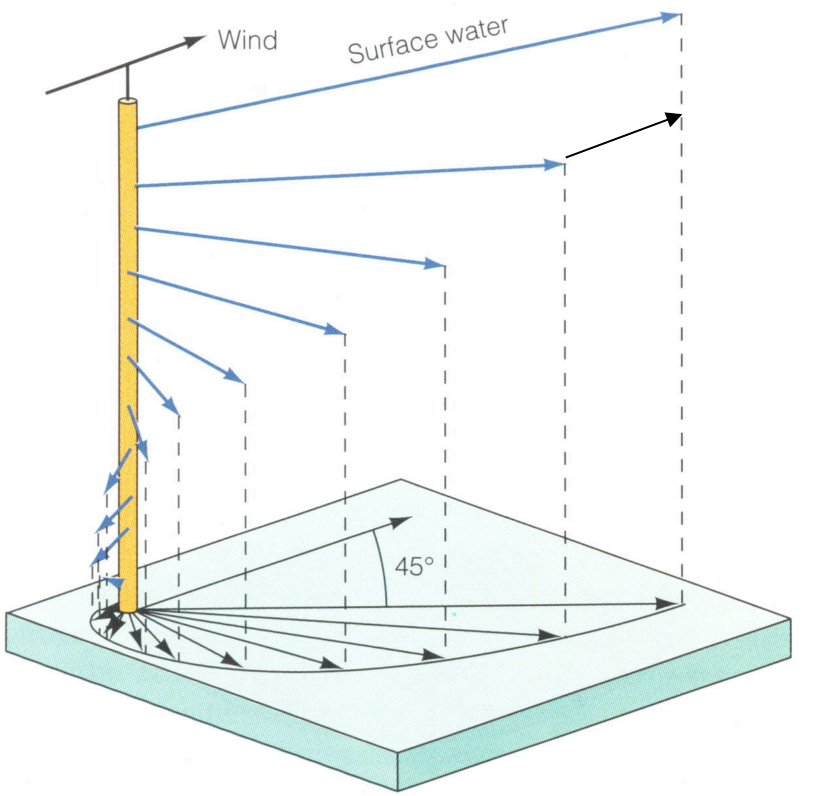

Figure from google image
http://www.eeb.ucla.edu/test/faculty/nezlin/PhysicalOceanography.htm

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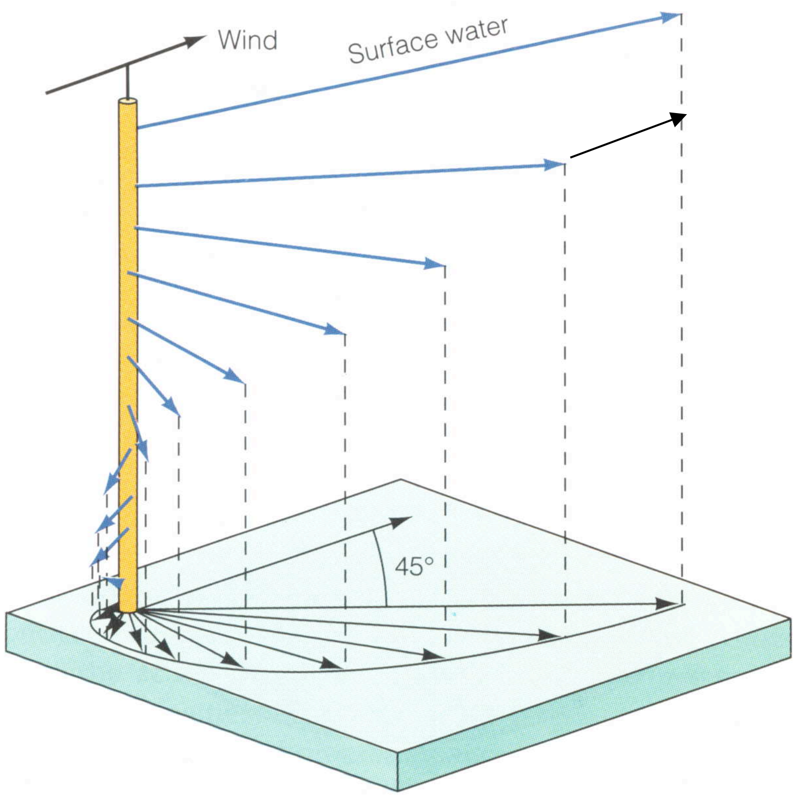
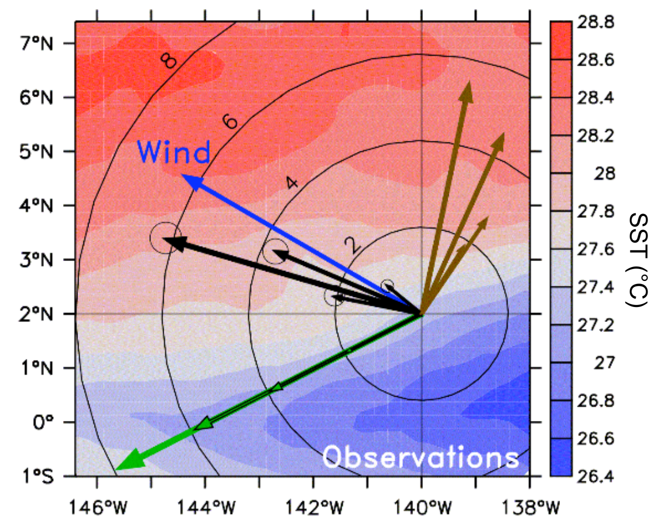




Geostrophic “thermal wind” shear is larger than the observed shear.

Figure from google image
http://www.eeb.ucla.edu/test/faculty/nezlin/PhysicalOceanography.htm

"Frontal Ekman Model" (Cronin and Kessler 2009):
Assume steady, linear flow; with uniform viscosity;
driven by wind stress at surface *and geostrophic flow at* $z = -H$*;*
in a front that is uniform with depth. Find $\boldsymbol{u_a}(z)$.

Equation of motion:

$$ifu = -\frac{1}{\rho}\nabla P + \nu \frac{\partial^2 u}{\partial z^2}$$

Boundary conditions:

$$at \;\; z = 0: \quad \frac{\partial u}{\partial z} = \frac{\tau_0}{\rho \nu}$$

$$at \;\; z = -H: \quad u = u_g$$

$$u = u_g + u_a, \quad where: \quad u_g = \frac{i}{\rho f}\nabla P \quad and \quad \frac{\partial u_g}{\partial z} = \frac{ig\alpha}{\rho f}\nabla T \equiv vertically\ uniform$$

“Frontal Ekman Model” (Cronin and Kessler 2009):

Assume steady, linear flow; with uniform viscosity;
driven by wind stress at surface *and geostrophic flow at $z = -H$;*
in a front that is uniform with depth. Find $\boldsymbol{u_a}(z)$.

Equation of motion:

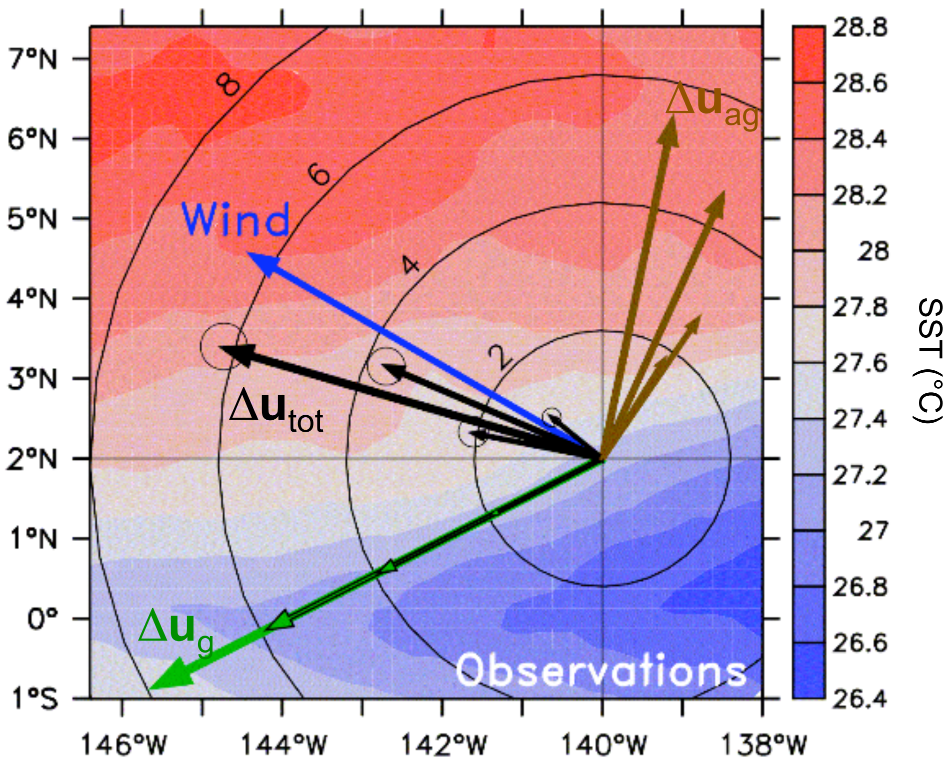
$$ifu = -\frac{1}{\rho}\nabla P + \nu\frac{\partial^2 u}{\partial z^2} \quad \rightarrow \quad ifu_a = \nu\frac{\partial^2 u_a}{\partial z^2}$$

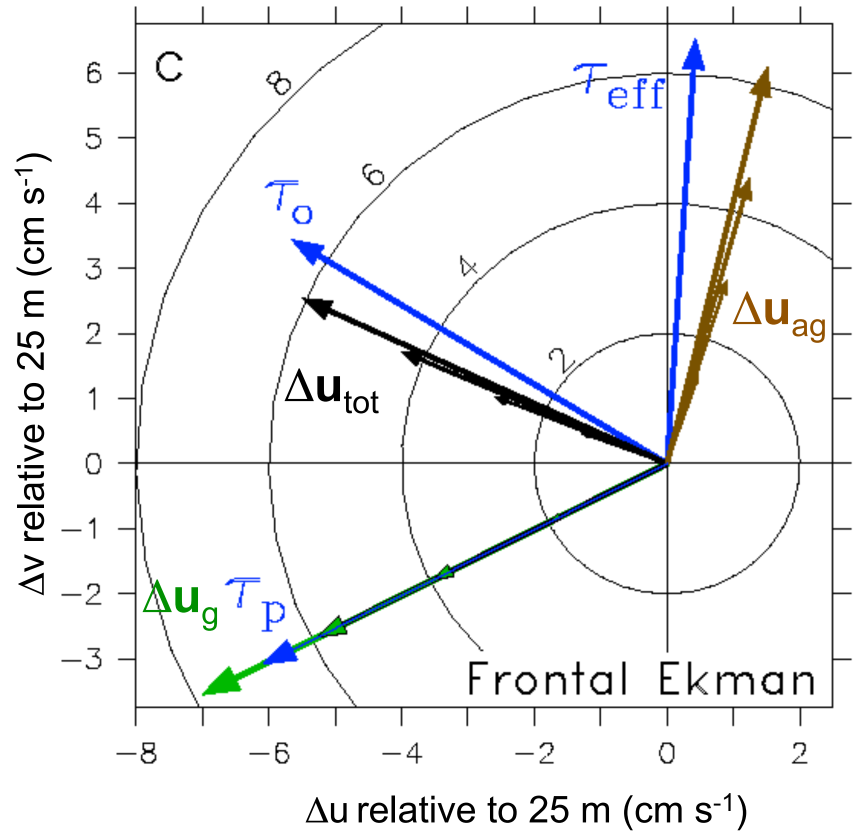
Boundary conditions: $\boldsymbol{u} = \boldsymbol{u_g} + \boldsymbol{u_a}$

$$at\ z = 0: \quad \frac{\partial u}{\partial z} = \frac{\tau_0}{\rho\nu} \quad \rightarrow \quad at\ z = 0: \quad \rho\nu\frac{\partial u_a}{\partial z} = \underbrace{\tau_0 - \rho\nu\frac{\partial u_g}{\partial z}}_{\tau_{eff}}$$

$$at\ z = -H: \quad u = u_g \quad \rightarrow \quad at\ z = -H: \quad u_a = 0$$

Ageostrophic Ekman Spiral is forced by the portion of wind stress that is out of balance with geostrophic shear: $\tau_{eff} = \tau_0 - \rho\nu\partial u_g/\partial z$

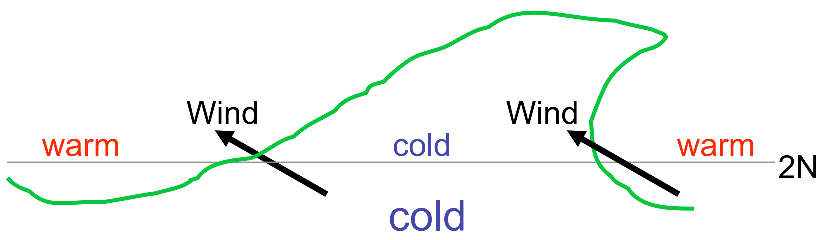




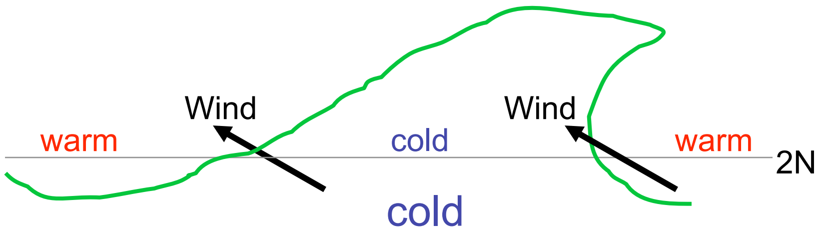
The ageostrophic Ekman response depends upon wind stress…
*and **strength & orientation of the front** relative to the wind:* $\tau_{eff} = \tau_0 - \tau_p$

The Ekman response is reduced when winds blow along a front.

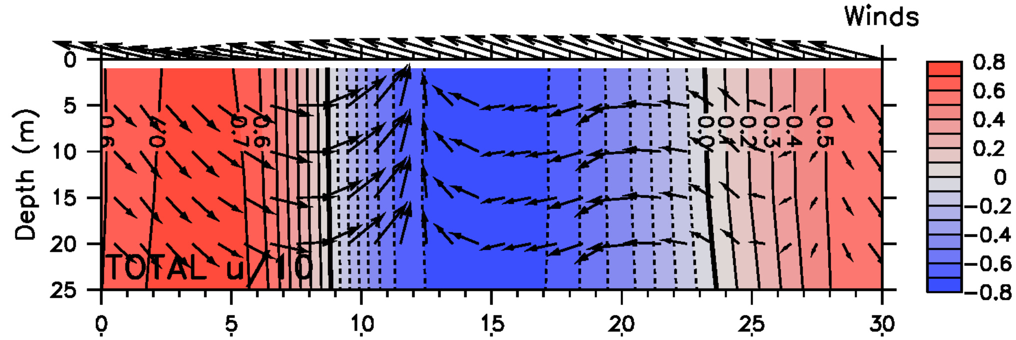
Tropical Instability Waves show how orientation of front relative to the wind affects ageostrophic shear

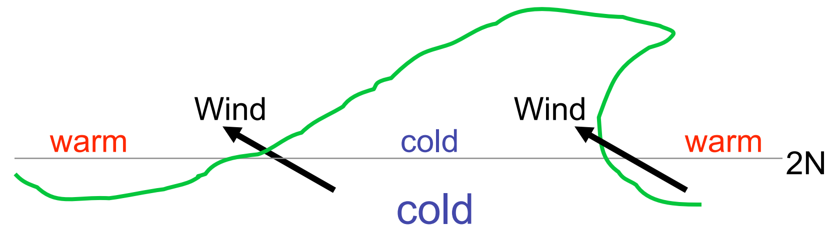


Tropical Instability Waves show how orientation of front relative to the wind affects ageostrophic shear



Composite TIW based on Obs





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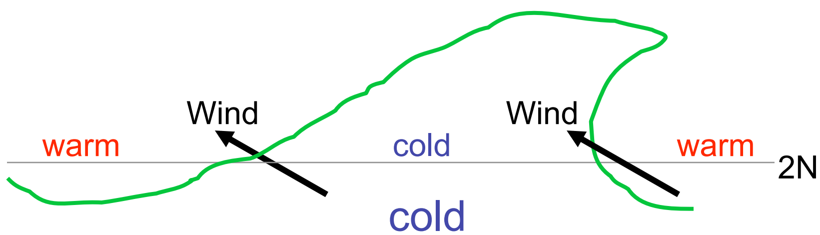

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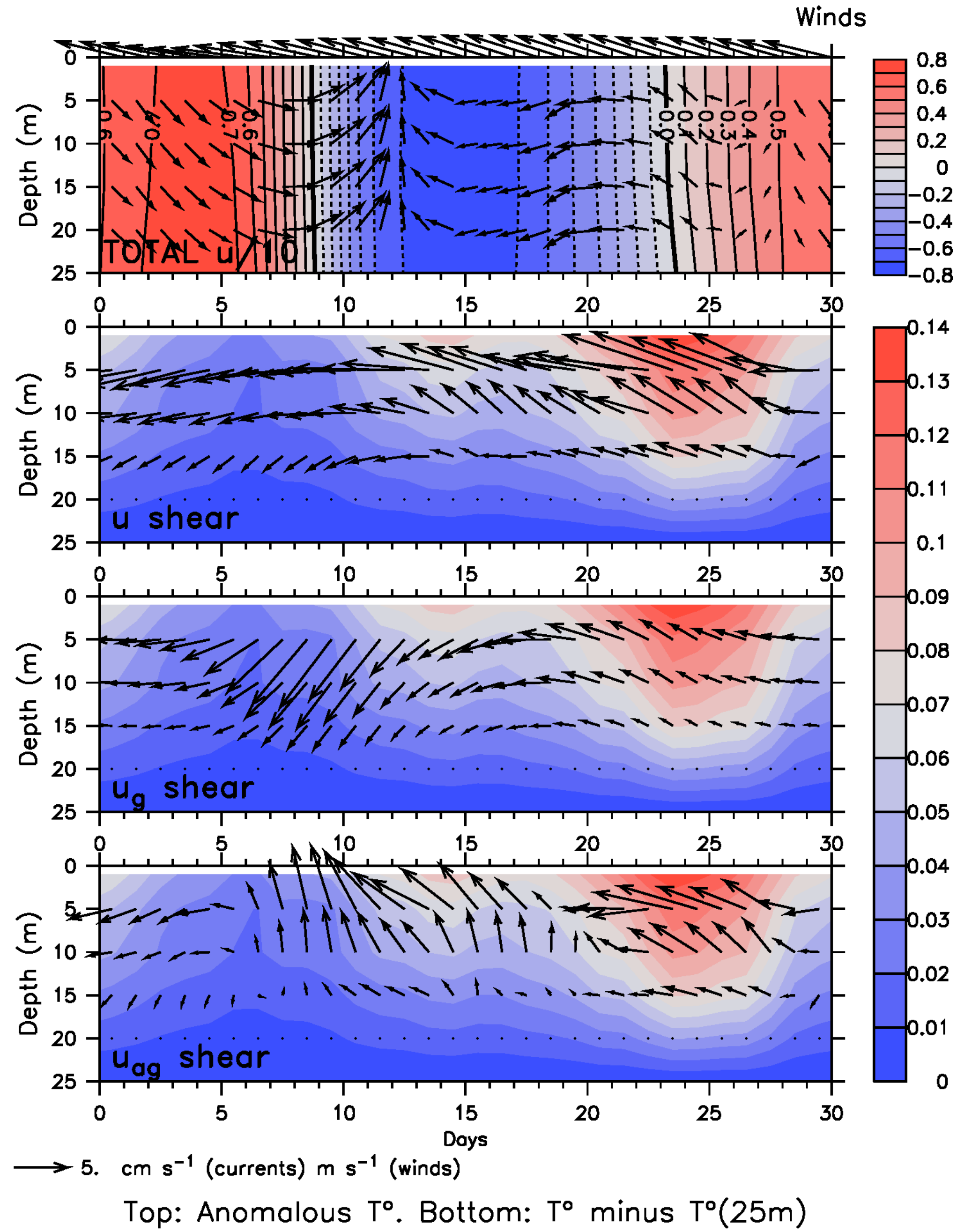
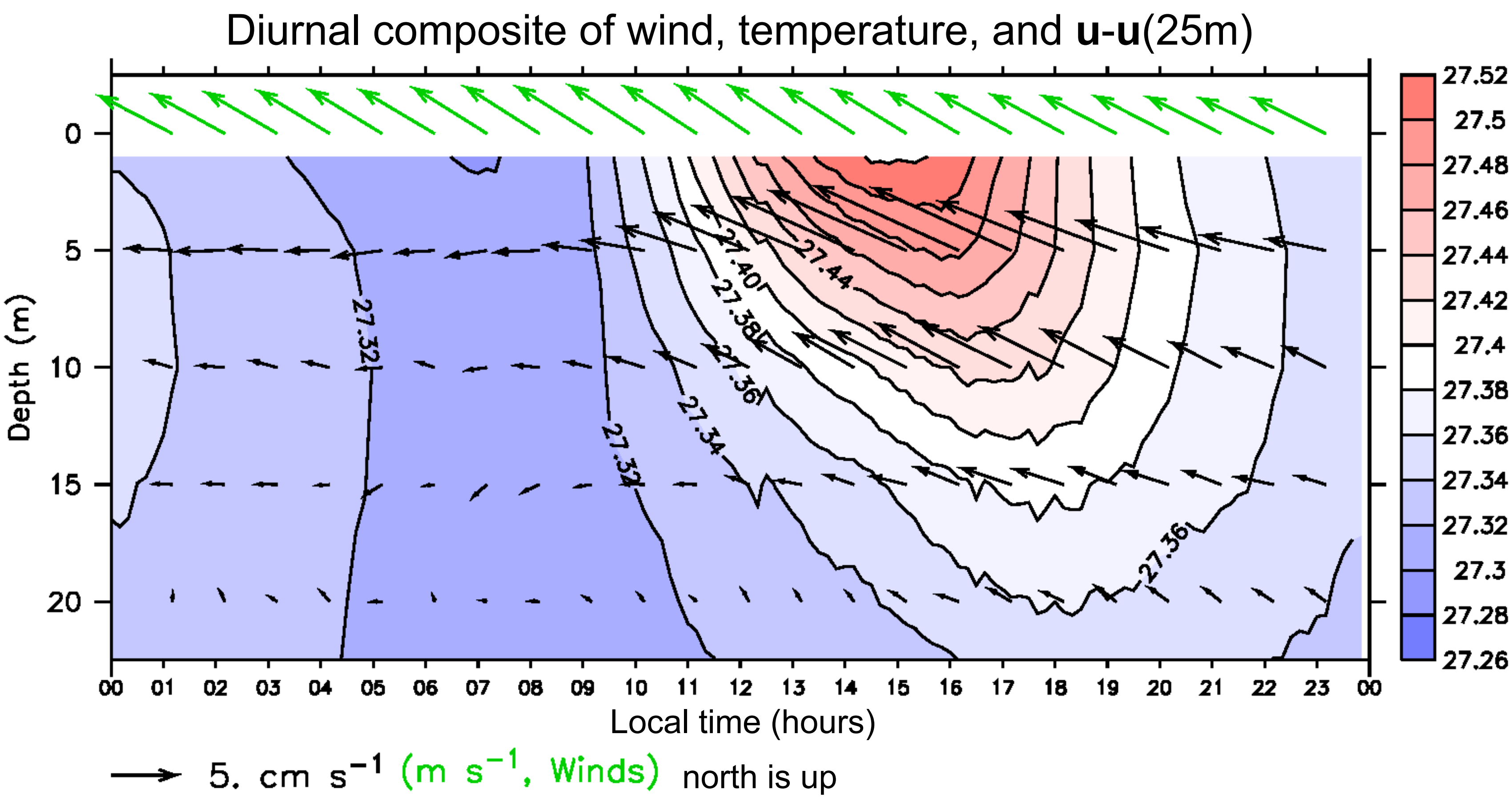


Strongest ageostrophic shear when wind blows across front

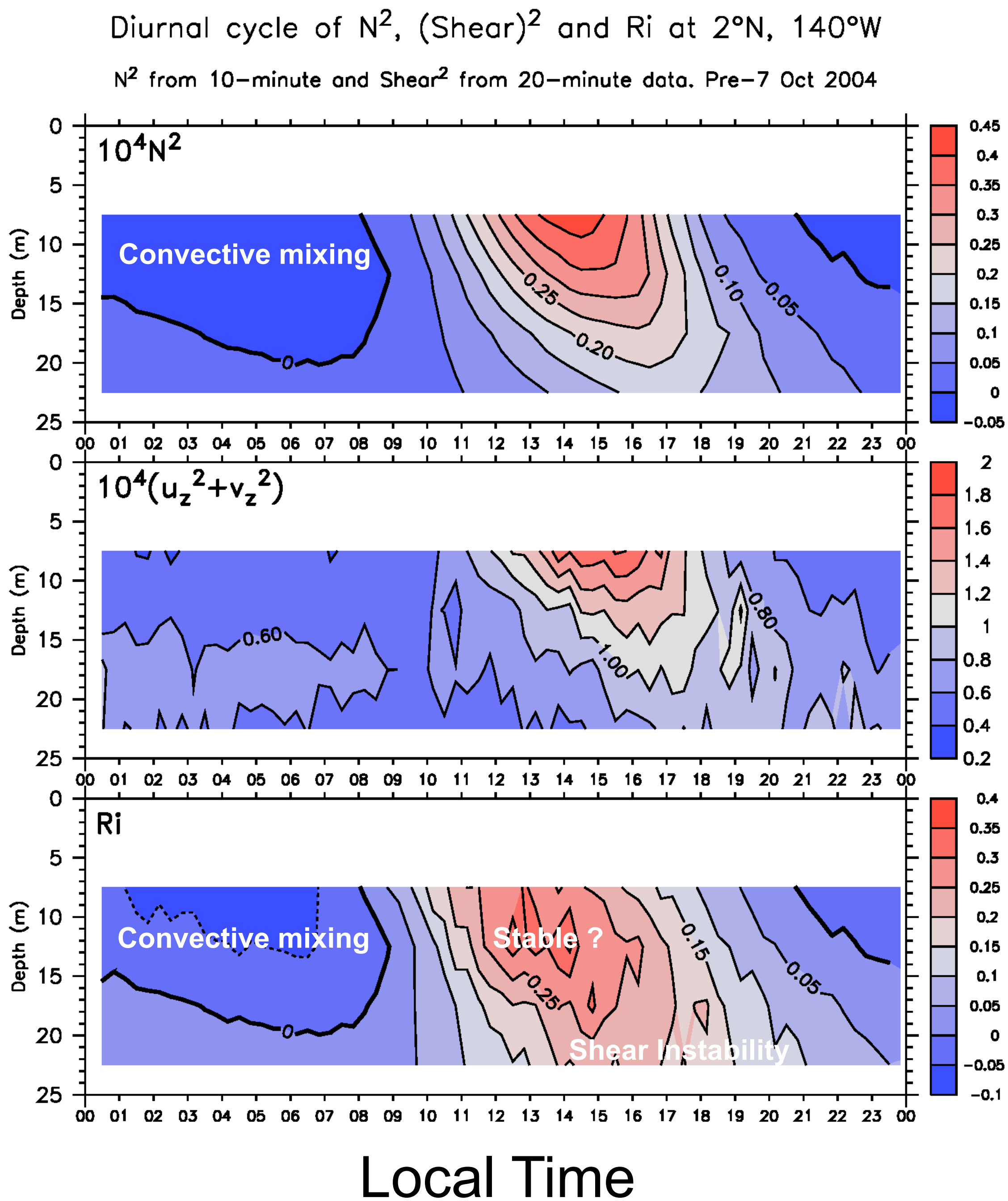
Weaker ageostrophic shear when winds are aligned with front.

Total shear stronger when stratification is stronger.



- *At 1600 local, currents at 5 m are 12 cm/s stronger than at 25m and are oriented in direction of wind. Nighttime shear is weak.*

- *Even weak daytime restratification can cause diurnal jet.*



- Convective mixing down to 10-20 m for 2100-0800 local.
- Ri>0.25 (stable?) near 10m for 1200-1500 local.
- Ri<0.25 (shear instability?) propagates downward?

Viscosity is likely to be larger in the upper 25 m than below due to both nighttime convective mixing and shear instability mixing due to the diurnal jet.

Summary (thus far)

- Wind stress balances the TOTAL surface shear, i.e., the combined geostrophic and ageostrophic shears.

  The ageostrophic Ekman spiral is forced by the portion of the wind stress that is out of balance with geostrophic shear.

- Shear is very sensitive to both the horizontal and vertical temperature distribution.

  Very weak daytime stratification (<0.2ºC/25m) resulted in a diurnal jet shear of 12 cm/s / 20m on average at 4 PM local!

What happens at the Equator *(where f = 0)* ?

$$u = u_g + u_a \qquad where: \quad u_g = -\frac{1}{\rho f}\nabla P$$

$$and \qquad \frac{\partial u_g}{\partial z} = \frac{g\alpha}{\rho f}\nabla T \quad (thermal\ wind)$$

What happens if viscosity ν is not constant?

$$\tau(z) = \rho\nu(z)\frac{\partial u}{\partial z}$$

What happens to the Ekman Transport at a front?

$$M_{ek} = -\frac{i\tau_0}{\rho_0 f} + \frac{i\nu}{f}\frac{\partial u}{\partial z}\bigg|_{z=-H}$$

Goal:
Develop a generalized Ekman model that is valid on the equator and can have a non-uniform viscosity.

(We are not the first to try this….)

"Stommel (1960) Model":

Assume steady, linear flow; with uniform density and viscosity; driven by wind stress at surface, and no stress (no shear) at $z = -H$. No flow through eastern and western boundaries. Find $P(x,y), \mathbf{u}(y,z)$.

Equation of motion:

$$ifu = -\frac{1}{\rho}\nabla P + \nu \frac{\partial^2 u}{\partial z^2}$$

Boundary conditions:

$$at\ \ z = 0: \quad \frac{\partial u}{\partial z} = \frac{\tau_0}{\rho \nu}$$

$$at\ \ z = -H: \quad \frac{\partial u}{\partial z} = 0$$

Good at center of EUC

No geostrophic / ageostrophic decomposition.

On equator, wind stress balances pressure gradient.

"Modified Stommel Model" used for OSCAR (Bonjean and Lagerloef 2002): Assume steady, linear flow; with uniform viscosity in a front that is uniform with depth; driven by wind stress at surface, and no shear at $z = -H$. Find $d\mathbf{u}/dz$ analytically.

Equation of motion:

$$\boxed{if \frac{\partial u}{\partial z} = -\nabla b} + \nu \frac{\partial^2}{\partial z^2}\left(\frac{\partial u}{\partial z}\right)$$

Boundary conditions:

$$at \;\; z = 0: \quad \frac{\partial u}{\partial z} = \frac{\tau_0}{\rho \nu}$$

$$at \;\; z = -H: \quad \frac{\partial u}{\partial z} = 0 \quad \rightarrow \quad \frac{\partial u_a}{\partial z} = -\frac{\partial u_g}{\partial z}$$

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Equation of motion:

$$if\frac{\partial u}{\partial z} = -\nabla b + \nu\frac{\partial^2}{\partial z^2}\left(\frac{\partial u}{\partial z}\right)$$

Boundary conditions:

$$at\ z = 0{:}\quad \frac{\partial u}{\partial z} = \frac{\tau_0}{\rho\nu}$$

$$at\ z = -H{:}\quad \frac{\partial u}{\partial z} = 0 \quad \rightarrow \quad \frac{\partial u_a}{\partial z} = -\frac{\partial u_g}{\partial z}$$

Realistic?

Perhaps a more realistic way to make $\tau_{-H} = 0$ is to have viscosity = 0 at the bottom of the viscous layer, rather than insisting that shear = 0 there.

"**Generalized Ekman Model**" (Cronin and Kessler 2009): Assume steady, linear flow; with prescribed viscosity that decays to *0* at depth $z = -H$; subject to prescribed buoyancy gradient and wind stress.
Find $\boldsymbol{\tau}(z)$: $\boldsymbol{\tau}(z) = \rho\nu\, d\mathbf{u}/dz$ numerically.

Equation of motion:
(vertical shear equation)

$$if\boldsymbol{\tau} = -\rho\nu\nabla b + \nu\frac{\partial^2\boldsymbol{\tau}}{\partial z^2}$$

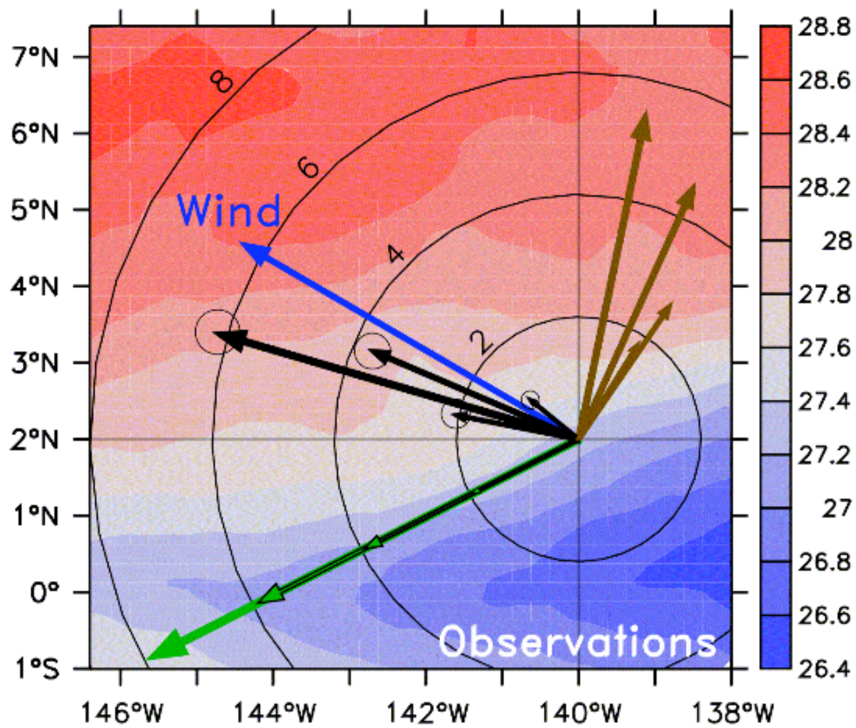
Boundary conditions:

$$at\ \ z = 0: \qquad \boldsymbol{\tau} = \boldsymbol{\tau}_0$$

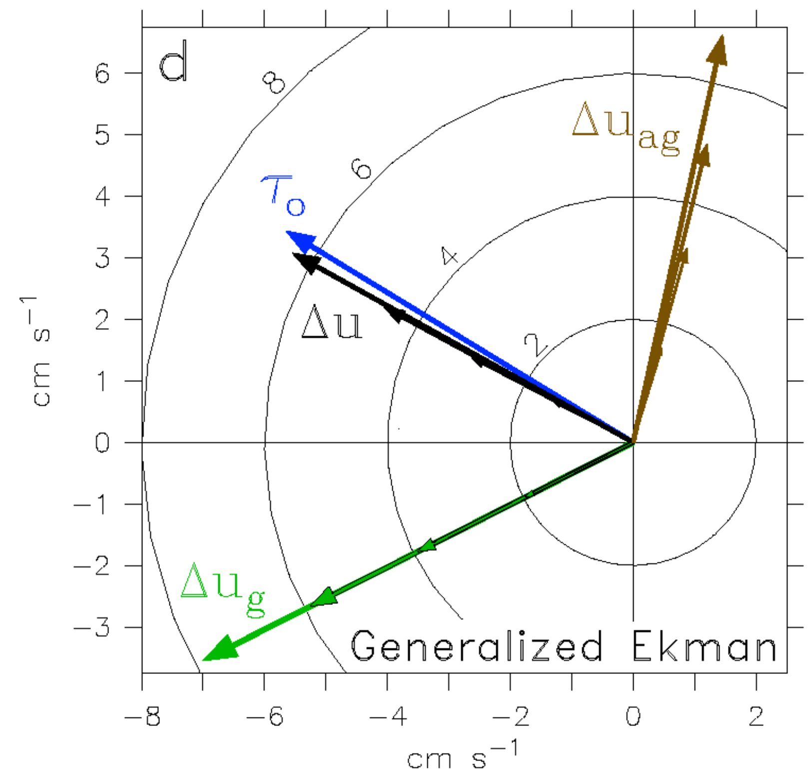
$$at\ \ z = -H: \qquad \boldsymbol{\tau} = 0 \quad (where\ \ \nu = 0)$$

As with the Classic Ekman model, ∇b and ν are prescribed. However, the generalized Ekman model is valid on the equator and in a frontal region, while the Classic Ekman model is not.

The Generalized Ekman model reproduces major features of the near-surface shear at 2ºN!



$$\nu = 0.02\left(e^{\left(\frac{z}{125m}\right)} - e^{\left(\frac{-250m}{125m}\right)}\right)$$ based on obs



Because our deepest measurement was at 25 m, we do not resolve the lower portion of the Ekman spiral where the frontal and generalized Ekman models are expected to differ.

Summary

- Wind stress balances the TOTAL surface shear, i.e., the combined geostrophic and ageostrophic shears.

  The ageostrophic Ekman spiral is forced by the portion of the wind stress that is out of balance with geostrophic shear.

- Shear is very sensitive to both the horizontal and vertical temperature distribution.

  Very weak daytime stratification (<0.2ºC/25m) resulted in a diurnal jet shear of 12 cm/s / 20m on average at 4 PM local!

- The effect of fronts on Ekman spiral is most pronounced at low latitudes.

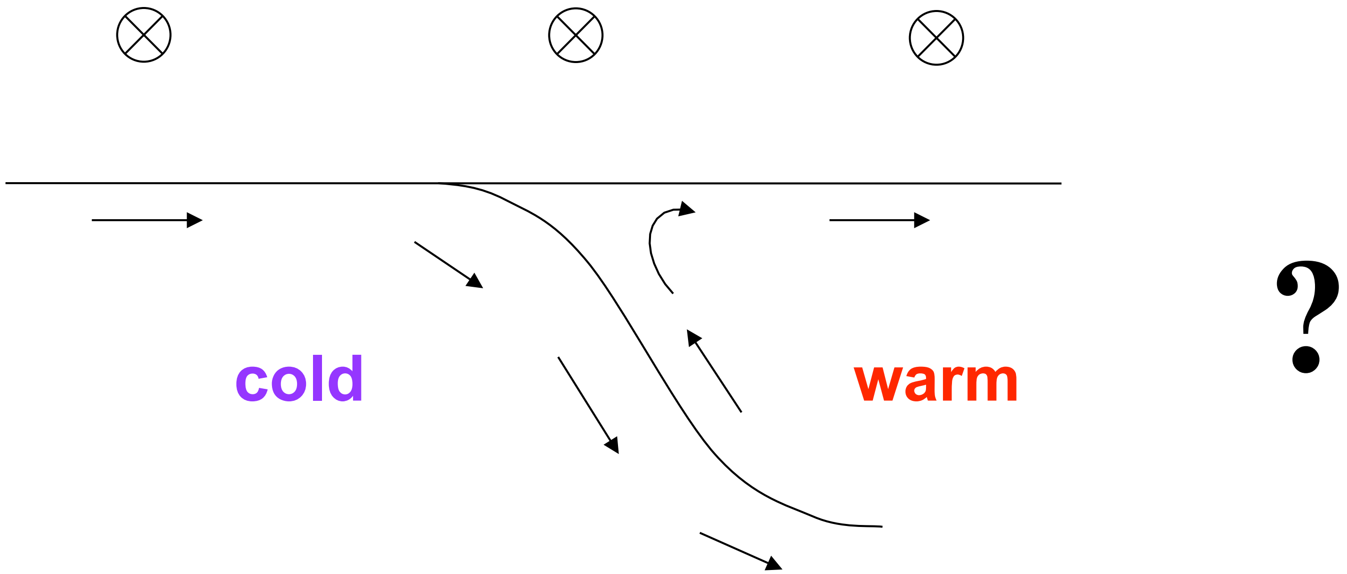
- A generalized Ekman model was developed that is valid on the equator and in frontal regions. It requires viscosity to be zero at the bottom of the viscous layer.

Consequences

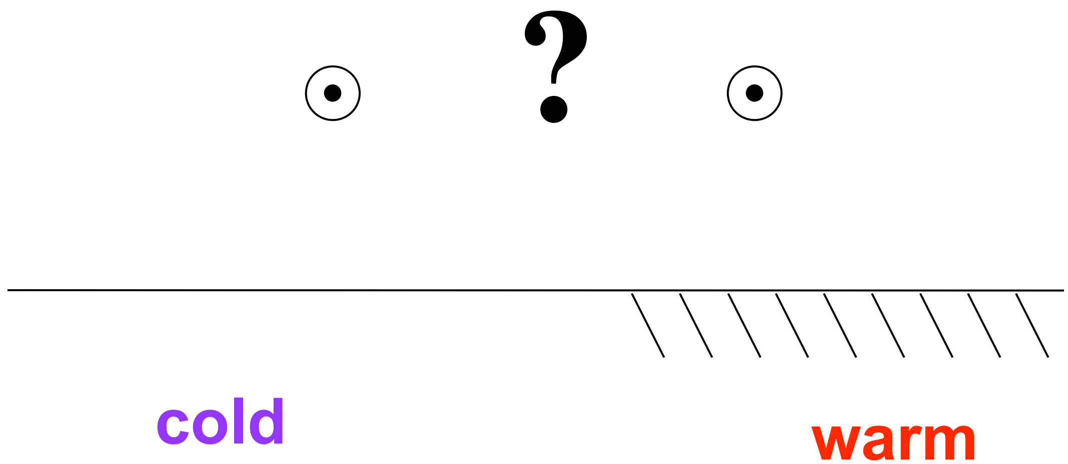
- In frontal region, near-surface Ekman flow is not necessarily to right of the wind stress.
- Boundary Conditions!

Ekman Transport in Frontal Region $= M_{ek} = -\frac{i\tau_0}{\rho_0 f} + \frac{i\nu}{f}\frac{\partial u}{\partial z}\Big|_{z=-H}$

- Traditional Ekman heat transport implicitly assume that viscosity decays with depth.
- At the center of the front, geostrophic shear & its Ekman transport (towards cold side) are largest. Thus there may be convergence & downwelling on cold side of front, and divergence & upwelling on the warm side.



Can Ekman transport advect heat?



Can these frontal Ekman dynamics be applied to the atmosphere?

Many thanks to…

- **the PMEL TAO and Engineering Development Division for assistance with this project.**

- **LuAnne Thompson (UW),** Thompson, L., *Ekman layers and two-dimensional frontogenesis in the upper ocean*. JGR, 2000.

- **Lief Thomas (Stanford),** Thomas, L. and C. Lee, Intensification of ocean fronts by down-front winds, JPO 2005**.**

- **Eric D’Asaro, RenChieh Lien, Fabrice Bonjean, Renellys Perez, ChuanLi Jiang, Andy Chiodi, Dennis Moore, and others, for enlightening discussions of this work.**