Alternative Solutions to the Classical Vertical Velocity Problem

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ABSTRACT

The kinematic method for determining vertical velocity ω in pressure coordinates is reviewed. Alternative objective procedures are derived for obtaining ω, and an analytical solution to the pressure-differentiated continuity equation is found. A variational formulation leads to a generalized objective adjustment for divergence estimates which yields improved, physically realistic estimates of ω. Case studies for intense mesoscale convection demonstrate the utility of an adjustment scheme based on the simplest hypothesis, namely, that the errors in divergence estimates are a linear function of pressure.

1. Introduction

A recurring problem in meteorology and oceanography is the estimation of the distribution of vertical velocity in the fluid. Routine measurements of vertical velocity are not taken. However, the vertical velocity pattern is usually inferred from the measurements of the horizontal velocity, pressure distribution and/or temperature distribution by one of several methods, commonly called 1) the kinematic method, 2) the adiabatic method, and 3) the omega equation (e.g., Petterssen, 1956; Haltiner *et al.*, 1963). The solution of the omega equation requires the specification of conditions at all boundaries. The adiabatic method requires good data on the temperature structure of the fluid. The kinematic method requires only boundary conditions at the top and/or bottom of a column within which we have estimates of the horizontal velocity divergence.

The purpose of this paper is to review the kinematic method in detail and suggest some new proposals for estimating the kinematic vertical velocity. It is my conjecture that many of the artificial techniques used to conjure up horizontal boundary conditions for the omega equation, such as cyclic boundaries, extrapolation away from the region of interest, etc., may not be appropriate or possible in many investigations. Since good estimates of the temperature field are not always available for the adiabatic technique, it becomes necessary to apply the kinematic method as a last resort.

The equation of continuity in pressure coordinates is

$$\frac{\partial\omega}{\partial p}+\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

where u and v, the horizontal velocity components, are measured relative to a constant pressure surface p. In oceanography, div$V$ vanishing would be appropriate. If we assume that we have an independent estimate of ω at the ground or lower reference pressure surface of interest, (1) may be integrated over a slice Δp of the atmosphere, i.e.,

$$\omega_p=\omega_{p+\Delta p}+\int_p^{p+\Delta p}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)dp. \tag{2}$$

If ω_0 is the specified value and $k=1(1)K$ denotes the levels at which we desire estimates of vertical velocity,

$$\left.\begin{aligned}\omega_k&=\omega_{k-1}+D_k\\ \omega_k&=\omega_0+\sum_{l=1}^{k}D_l\end{aligned}\right\}, \tag{3}$$

where D_k is the pressure-weighted, mean horizontal divergence for the slice Δp of the column. We can estimate D_k from observations of u, v and thus, in principle, we can determine ω at any higher level in the column

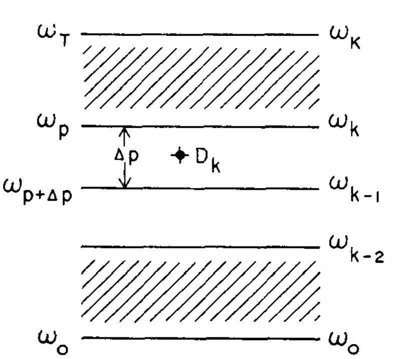

FIG. 1. Schematic of an arbitrary pressure level k. The vertical velocities ω are separated by a pressure increment Δp; D_k is the pressure-weighted, mean horizontal divergence for the slice, Δp; ω_0 and ω_T are the known estimates of ω at the bottom and top of the column; and ω_k are the objectively obtained estimates of ω_p.

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(see Fig. 1). However, it is well known that the ω_k become successively less acceptable as k increases, due to errors in the estimates of D_k. The value of ω at the top of any column usually is found to be either too low or too high when compared with independent, physically realistic estimates. It is my purpose to describe a modification of (3) which may be used to obtain physically acceptable values of ω throughout the column.

2. Second-order adjustment

Some authors (e.g., Lateef, 1967) have solved the discrepancy by forcing ω to be zero or some prescribed value ω_T at the top of any column. A non-zero estimate for ω_T might be the adiabatic ω. Eq. (1) is solved after first differentiating with respect to pressure to obtain

$$\frac{\partial^2\omega}{\partial p^2}=-\frac{\partial}{\partial p}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right). \tag{4}$$

This problem is now second-order in ω, and two boundary conditions can be applied. We note, however, that we are no longer assured that (1) is satisfied at every point in the fluid.

If Δp is constant for each slice, (4) becomes

$$-\omega_{k+1}'+2\omega_k'-\omega_{k-1}'=D_{k-1}-D_k, \quad k=1(1)K, \tag{5}$$

which is a set of K-coupled unknowns. We might use the well-known tridiagonal algorithm to effect a solution of (5); however, in this special case we can actually find the solution analytically and gain valuable physical insight about the adjustment we are applying to the atmospheric data.

If we view (5) as a matrix problem, $\mathbf{AW}=\mathbf{B}$, $\mathbf{A}$ is tridiagonal with diagonal element 2, and off-diagonal elements -1. It is easily shown (O'Brien, 1969) that its inverse, $\mathbf{A}^{-1}=[a_{ij}']$, is symmetric and has the form

$$a_{ij}'=j(k+1-i)/(k+1), \quad \text{if } j\leq i. \tag{6}$$

The solution to (5) is $\mathbf{A}^{-1}\mathbf{B}=\mathbf{W}$, where $\mathbf{W}^T=[\omega_1', \omega_2' \ldots, \omega_k']$ and $\mathbf{B}$ is the right-hand side of (5). After some algebra it can be shown that

$$\omega_k'=\omega_k-\frac{k}{K}(\omega_K-\omega_T), \tag{7}$$

where ω_k is the value obtained from (3), the usual kinematic estimate. The two boundary conditions are satisfied by (7), i.e., at $k=0$, $\omega'=\omega_0$, and at $k=K$, $\omega'=\omega_T$. We observe that, physically, the application of the second-order equation (4) in preference to (1), implies that we are changing the vertical velocity at any level k by an amount which linearly depends on the distance from the lowest surface. The "error" we observe in ω at the top of the column is $(\omega_K-\omega_T)$. This "error" is distributed linearly throughout the column. However, the "error" is not in ω but in our estimates of the pressure-weighted divergence D_k. What does this imply about the "correction" to the divergence field?

If we write (7) for k and $k-1$ and subtract, we obtain

$$D_k'=D_k-(\omega_K-\omega_T)/K, \tag{8}$$

where D_k' can be considered a corrected D_k. Note that the correction, $(\omega_K-\omega_T)/K$, is independent of k. As Lateef points out, the second-order adjustment scheme is equivalent to adjusting the divergence at every level by a constant. In the case of evenly spaced data, the divergence is adjusted by the amount $(\omega_K-\omega_T)/(\Delta p\, K)$ at every level. This physical understanding is important since, clearly, there must be a better correction hypothesis than a uniform adjustment of the divergence estimates!

3. Variational formulation

Consider the general problem of determining kinematic velocities. Let us presume that we have I estimates, d_i, of horizontal divergence at discrete positions p_i. Assume also that we have J estimates of omega, ω_j, at levels p_j. The coordinate levels, p_i and p_j, need not be uniformly spaced; in fact, we expect $p_i\neq p_j$. (In practice, it may be useful to smooth the vertical distributions of horizontal divergence by applying objective or subjective techniques and employ evenly spaced d_i.) We wish to find objectively adjusted values of omega ω_j' and divergence d_i', which satisfy certain physical constraints. Associated with each datum is an error variance, σ_i^2 or σ_j^2, of the variable.

The formulation of the problem follows Sasaki (1958) and Stephens (1965). Let

$$F_l\left(f_i, \frac{\partial f_i}{\partial p}, \frac{\partial^2 f_i}{\partial p^2}, \cdots, \frac{\partial^n f_i}{\partial p^n}\right)=0 \tag{9}$$

represent a set of constraints desired for certain initial estimate fields f_i. Let the ith objectively modified variable, $f_i=f_i(\mathrm{p})$, be continuous along with its derivatives through order $2n$ in a region μ and take on prescribed values on S the boundary of μ. Let a difference functional be defined by

$$E(f_i')=\int_\nu\left[\sum_i \kappa_i(f_i'-f_i)^2+\sum_j \Gamma_j F_j\right]d\nu, \tag{10}$$

where Γ_j are Lagrangian multipliers and κ_i are the Gauss precision moduli. These latter are defined (Whittaker and Robinson, 1944) by

$$\kappa_i=\frac{1}{2\sigma_i^2}. \tag{11}$$

The objectively modified values are determined by requiring the first variation of E to vanish.

In the present problem, f_i represents both d_i and ω_j. The principal constraint is the continuity equation (1). Anticipating knowledge of the vertical velocity at the top and bottom of any column, i.e., ω_0, ω_T, we may also require that these two conditions be constraints on the data.

The Gauss precision moduli must be specified as functions of p, d_i', ω_i', which represent relative weights for each datum. Inherent in (10) is all our data and physical knowledge of the state of the fluid. We may make the adjustment as simple or complicated as desired. For example, κ_i might be zero, constant, a linear function of pressure, or nonlinear; they might be chosen to be a function of rawinsonde distance from station, a function of turbulence intensities at some heights, etc. There are no restrictions whatsoever on choice of κ. Additional constraints might be that the maximum or minimum omega must exist at some specified level.

The remainder of this paper is concerned with a few formulations that have been successfully used.

Let us assume that the Gauss precision moduli are independent of d_i and ω_j. However, they may depend on pressure and any other parameter (such as turbulence intensity, for example). The functional E becomes

$$E(\omega_j', d_i', \Gamma_l)=\sum_{i=1}^{I} \kappa_i(d_i'-d_i)^2+\sum_{j=1}^{J} \kappa_j(\omega_j'-\omega_j)^2 +\sum_{l=1}^{3} \Gamma_l F_l, \quad (12)$$

where F_1 is a finite difference analogue of (1), $F_2=\omega(\text{bottom})-\omega_0$, and $F_3=\omega(\text{top})-\omega_T$. The adjusted values are found by setting the first variation of E to be zero. The resultant equations are a set of coupled linear equations with unknowns ω_j', d_i', Γ_l if κ_i and κ_j are independent of ω and d. Otherwise they are a nonlinear set of equations which can be solved by standard iteration procedures. The specific use of this variational formulation is demonstrated in the next section.

4. Generalized adjustment criteria

Consider the usual problem confronting the analyst. He has available estimates of pressure-weighted horizontal divergence D_i and independent estimates of the vertical velocity ω near the ground and at some great height in the atmosphere. However, we will assume that he has no *a priori* estimates of ω at any intermediate level. For the sake of simplicity we shall assume that the divergence is given at equally spaced pressure intervals. The error variances σ^2 are presupposed to be independent of D and ω but depend in some specified way on pressure p and some external parameters. The variational functional (12) can be written as

$$\left.\begin{aligned} E(D_i', \lambda)&=\sum_{i=1}^{K} \kappa_i(D_i'-D_i)^2+2\lambda F \\ F&=\sum_{k=1}^{K} D_k'+(\omega_0-\omega_T) \end{aligned}\right\}, \quad (13)$$

where ω_0 and ω_T are the specified vertical velocities at the bottom and top of the layer. The equations to be solved are

$$\left.\begin{aligned} D_k'-D_k&=-\lambda/\kappa_k,\ k=1(1)K \\ \sum_{k=1}^{K} D_i'&=-(\omega_0-\omega_T) \end{aligned}\right\}. \quad (14)$$

The solution is

$$D_k'=D_k-\frac{\sigma_k^2}{\sum_{i=1}^{K}\sigma_i^2}\left(-\omega_0+\omega_T-\sum_{i=1}^{K} D_i\right), \quad (15)$$

where (11) has been used. The vertical velocity is given by

$$\omega_k'=\omega_k-\frac{S_k}{S_K}(\omega_K-\omega_T), \quad (16)$$

where ω_k and ω_K are given by (3) and,

$$S_k=\sum_{i=1}^{k}\sigma_i^2. \quad (17)$$

The equations (15)–(17) are used to calculate the objectively adjusted divergence and vertical velocity. It is instructive to look at some special examples.

5. Special adjustment criteria

Consider the case when σ_i is independent of pressure, i.e., a constant; (15) reduces to (8) and (16) to (7). In other words, the "error" is distributed uniformly over all divergence estimates.

Consider the case when σ_i^2 is a linear function of k or pressure; (15) and (16) may then be written as

$$D_k'=D_k-\frac{k}{M}(\omega_K-\omega_T), \quad (18)$$

$$\omega_k'=\omega_k-(\omega_K-\omega_T)\left[\frac{k(k+1)}{K(K+1)}\right]. \quad (19)$$

Why would we consider these adjustments? It is well known and documented (Duvedal, 1962) that wind measurements from a GMD-1A system deteriorate with decreasing *elevation angles* which may be the combined consequence of strong winds and sounding duration (indirectly altitude). If we use (18) and (19), the correction is a linear function of the pressure. Near the ground the correction is essentially zero; the maximum correction occurs high in the atmosphere. The weighted correction for ω is now almost quadratic, with essentially no correction in the lower part of the atmosphere and a large correction in the upper atmosphere. Note that the boundary conditions are satisfied.

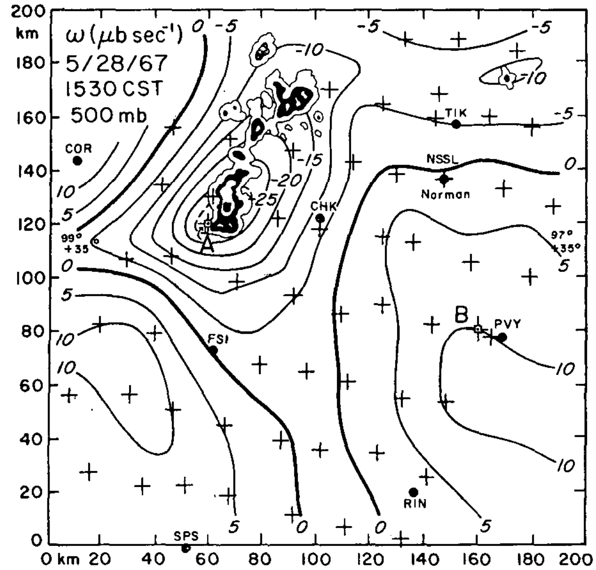

FIG. 2. Vertical motion [μb sec⁻¹] at 500 mb at 1530 CST 28 May 1967 as obtained from (3). Intensity contours of PPI radar echo configuration are shown. The vertical profiles for grid points A and B are shown in Figs. 3 and 4.

This adjustment procedure has been applied in several actual atmospheric problems, and has given very acceptable results. An example is shown from Fankhauser (1969) in Figs. 2–5. This study was concerned with a mesoscale convective regime in Oklahoma, where several strong, active thunderstorms developed within a network of rawinsonde stations. The carefully analyzed data gave poor estimates of the kinematic vertical velocity at 100 mb. The choice of $\omega_T=0$ or the independent adiabatic value gave much more physically realistic results.

Fig. 2 shows the network of stations and the area of maximum convection for a particular time as indicated by radar echo intensity contours. Figs. 3 and 4 show the vertical profiles of divergence, kinematic vertical velocity [Eq. (3)], and the adjusted profiles [Eqs. (18), (19)] for two selected grid points (A and B) indicated on Fig. 2. Note that the adjustment in the heavy convective case (A) is appreciable. For the column outside of the convective regime, the change is slight. The maximum adjustment of the divergence profiles is equivalent to a wind speed discrepancy of 1 m sec⁻¹.

As noted, the overall adjustment to ω demonstrated in Figs. 3 and 4 is appreciable; however, the distribution of ω' shown in Fig. 5 is relatively unchanged and ω' continues to effectively mirror the convective regime as seen by radar. Using energy aspects associated with latent heat release, Fankhauser has thus demonstrated that ω' is a physically improved evaluation of the vertical motion in his case.

Our studies indicate that the objective adjustment given by (18) and (19) yields very significant physical improvement in the vertical velocity estimates for the entire column. The improvement is judged by the correlation of convective intensity, as measured by radar, with the resultant "corrected" vertical velocity distributions for *all* atmospheric levels in many case studies.

The physical insight gained by studying the results obtained in the previous sections enables us to consider alternative adjustment procedures. Suppose, for example, we were confident that the errors in horizontal divergence estimates were directly related to the magnitude of the wind velocity at a particular level, i.e., if the wind speed V is large, we would expect a larger error in our divergence estimate. It appears that there are many physical situations under which this would be very likely. Duvedal (1962) documents this possibility. From our previous results we relate σ_i^2 to V_i and write "corrected" divergence and vertical velocity fields as

$$D_k'=D_k-\frac{V_k}{Q_K}(\omega_K-\omega_T), \tag{20}$$

$$\omega_k'=\omega_k-\frac{Q_k}{Q_K}(\omega_K-\omega_T), \tag{21}$$

where V_k and D_k are the measured wind speed and pressure-weighted divergence at level k, ω_k' is given by (3), and

$$Q_k=\sum_{l=0}^{k} V_l. \tag{22}$$

Under this hypothesis, the divergence D_k is corrected by a factor proportional to the wind speed at level k. Note that the vertical velocity ω_k is corrected by a factor Q_k which is the sum of the wind speeds for every level below k.

In general, if we conclude the divergence to be in error due to any criterion f which can depend in any

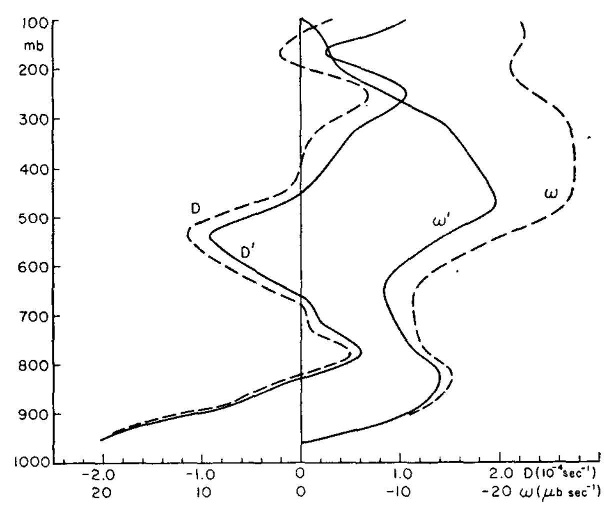

FIG. 3. Vertical profiles of unadjusted (dashed) D and ω from Eq. (3) and objectively adjusted (solid) D' and ω' from Eqs. (18) and (19), for grid point A near strongest convection.

way on the physical or geometrical properties of the fluid configuration, then the best estimates of the actual divergence and vertical velocity, if we know ω_0 and ω_T, are given by

$$D_k' = D_k - \frac{f_k}{R_K}(\omega_K - \omega_T), \quad (23)$$

$$\omega_k' = \omega_k - \frac{R_k}{R_K}(\omega_K - \omega_T), \quad (24)$$

where

$$R_k = \sum_{\alpha=0}^{k} f_\alpha. \quad (25)$$

These latter formulas enable us to construct objectively consistent profiles of D_k' and ω_k' under any hypothesis available. The resultant ω will be consistent with the principle of conservation of mass and will satisfy the boundary conditions at the top and bottom of any column.

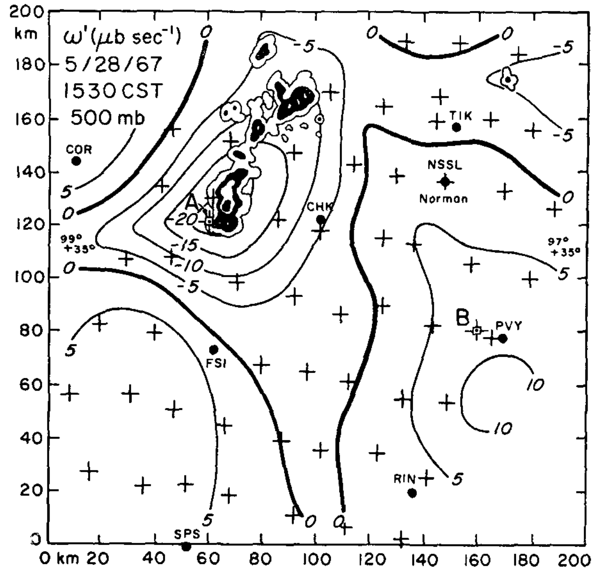

FIG. 5. Objectively adjusted vertical motion [μb sec^{-1}] at 500 mb at 1530 CST 28 May 1967 as obtained from Eq. (19).

6. The global adjustment scheme

In many severe convection studies, there is no assurance that ω should be zero at the top of any column, say at the 100-mb level. However, over a very large region the average ω should be zero by conservation of mass. An objective technique is derived which can determine ω from (3), with the added constraint that the mean ω at the top is a predetermined value (usually zero). In this case the boundary condition is "global" in the mathematical sense. At the top of any one column, ω is not specified, but over the entire region of interest ω_T has a prescribed mean.

The basic procedure is to apply a Lagrangian multiplier, e.g., Gruber and O'Brien (1968), to force the numerical result to conform to the global boundary condition.

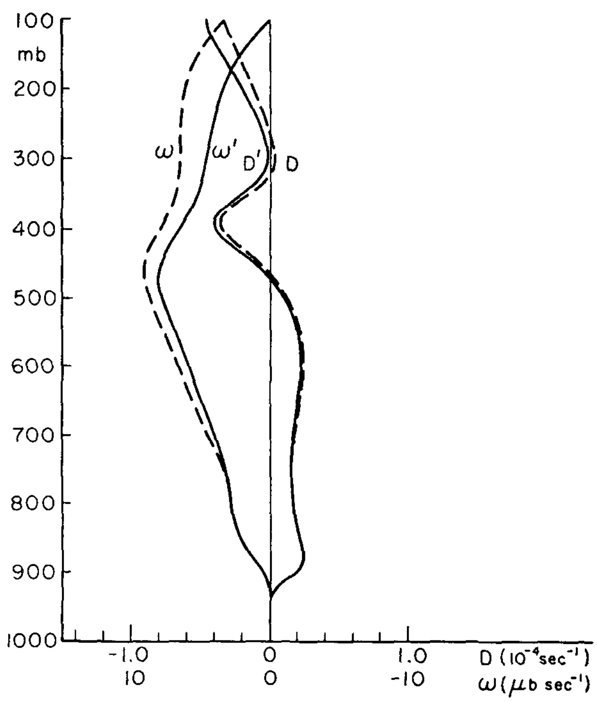

FIG. 4. Same as Fig. 3 except for grid point B in subsiding air.

Let us construct the function

$$S(\omega_{ijk}, \lambda) = \sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{k=1}^{K} (\omega_{ijk} - \omega_{ij,\,k-1} + D_{ijk})^2 + \lambda\Big(\sum_{i=1}^{N} \sum_{j=1}^{M} \omega_{ijK} - \alpha\Big). \quad (26)$$

In (26), $\omega_{i,j,k}$ is the value at the point (i,j,k); $k=K$ is the top of the column under study; $\omega_{i,j,o}$ is prescribed; λ is the Lagrangian multiplier; and α is the preset value of mean vertical velocity (usually zero) at the top of the model atmosphere. It is important to realize that if the quantities in the two parentheses are zero, then (3) is satisfied everywhere and the mean velocity at the top has the prescribed value α. If we minimize S, we will obtain an estimate that is as close as possible to the data, $D_{i,j,k}$.

We differentiate S with respect to its variables, $\omega_{i,j,k}$ and λ to obtain

$$\frac{\partial S}{\partial \omega_{lrq}} = 2(\omega_{lrq} - \omega_{lr,\,q-1} + D_{lrq}) - 2(\omega_{lr,\,q+1} - \omega_{lrq} + D_{lr,\,q+1});$$

$$l=1,N;\ r=1,M;\ q=1,\ K-1, \quad (27)$$

$$\frac{\partial S}{\partial \omega_{lrK}}=2(\omega_{lrK}-\omega_{lr,\,K-1}+D_{lrK})+\lambda, \tag{28}$$

$$\frac{\partial S}{\partial \lambda}=\sum_{i=1}^{N}\sum_{j=1}^{M}\omega_{ijK}-\alpha. \tag{29}$$

In these equations, there are NMK unknowns $\omega_{i,j,k}+\lambda$, i.e., (27) is $NM(K-1)$ equations, (28) is NM equations, and (29) is one equation.

If we let (27)–(29) equal zero, we have a set of $NMK+1$ equations with $NMK+1$ unknowns. This set, unfortunately, is not easily solved with the usual matrix subroutines because the number of equations will be very large in most practical cases. However, this difficulty can be obviated.

Let us consider the equations in matrix form, i.e.,

$$\mathbf{AW}=\mathbf{B}, \tag{30}$$

where

$$\mathbf{W}^T=[\omega_{111},\omega_{112},\ldots,\omega_{11k},\omega_{121},\ldots,\omega_{NMK},\lambda]; \tag{31}$$

$\mathbf{B}$ contains the terms including $D_{i,j,k}$ for each equation, but whose last element is zero. We can now write $\mathbf{A}$ as

$$\mathbf{A}=\begin{bmatrix}\mathbf{C}_{11} & \varphi & \cdot & \cdot & \cdot & \Gamma\\ \varphi & \mathbf{C}_{12} & & & \cdot & \cdot\\ \varphi & & & & \cdot & \cdot\\ \cdot & & & & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & \mathbf{C}_{NM} & \Gamma\\ \Gamma^T & \Gamma^T & & & \Gamma^T & 0\end{bmatrix}.$$

The square matrices, $\mathbf{C}_{ij}$, are tridiagonal and identical, and of order K; thus

$$\mathbf{C}_{ij}=2\begin{bmatrix}2 & -1 & 0 & \cdot & \cdot & 0\\ -1 & 2 & 0 & & & \cdot\\ 0 & & & & & \cdot\\ \cdot & & & & & \cdot\\ \cdot & & & & 2 & -1\\ 0 & \cdot & \cdot & \cdot & -1 & 1\end{bmatrix}, \tag{32}$$

and

$$\Gamma^T=[0,0,\ldots,1], \tag{33}$$

where Γ is of order K. Note that $\mathbf{C}_{ij}$ is symmetric and independent of i,j. It can be easily shown that the inverse of $\mathbf{C}$ is symmetric and is given by

$$\left.\begin{aligned}\mathbf{C}^{-1}&=[c'_{rs}]\\ c_{rs}'&=\tfrac{1}{2}r,\quad \text{if}\quad r\le s\end{aligned}\right\}. \tag{34}$$

We then define a new matrix $\mathbf{Q}$ which is block-diagonal, with diagonal element $\mathbf{C}^{-1}$ except for $q_{NMK+1,\,NMK+1}$, which is unity. $\mathbf{Q}$ is thus of the form

$$\mathbf{QA}=\begin{bmatrix}\mathbf{I} & \varphi & \varphi & \cdot & \cdot & \cdot & \mathbf{J}\\ \varphi & \mathbf{I} & \varphi & \cdot & \cdot & \cdot & \mathbf{J}\\ \vdots & & & & & & \\ \varphi & & & & & \mathbf{I} & \mathbf{J}\\ \Gamma^T & \Gamma^T & \cdot & \cdot & \cdot & \Gamma^T & 0\end{bmatrix}, \tag{35}$$

and is completely inverted in the Gaussian sense except that Γ^T is in the lowest row. $\mathbf{J}^T=\frac{1}{2}[1,2,3,\ldots,K]$, $\mathbf{I}$ is the identity matrix and φ is the null matrix.

If we apply Gaussian elimination to the last row, we have

$$\mathbf{QA}\rightarrow\mathbf{Q}'=\begin{bmatrix}\mathbf{I} & \varphi & \varphi & \cdot & \cdot & \cdot & \mathbf{J}\\ \varphi & \mathbf{I} & & & & & \\ \vdots & & & & & & \vdots\\ \varphi & & & & & \mathbf{I} & \mathbf{J}\\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot & -NMK\end{bmatrix}, \tag{36}$$

where $\mathbf{Q}'$ is upper triangular and is easily solved by back substitution if we also alter the right-hand side of $\mathbf{AW}=\mathbf{B}$.

With some rearrangement, it can be shown that the estimate of $\omega_{i,j,k}$ is given by

$$\omega_{ijk}'=\omega_{ijk}-\frac{k}{NMK}\Big(\sum_{i=1}^{N}\sum_{j=1}^{M}\omega_{ijK}-\alpha\Big). \tag{37}$$

Again we observe that, physically, we are applying a linear-weighted correction to ω. Also, the stated boundary conditions are met, i.e., $\omega'=\omega_0$, and, if we sum over (i,j) for $k=K$, $\omega'=\alpha$ at the top. Clearly, the corrections to ω in (37) are quite small since, in general, we expect NMK to be large. If we find at level K that the patterns of $\omega_{i,j,K}$ are intense (several maxima and minima), we would expect ω_{ijk}' to reflect the same patterns. Only the mean ω will be changed at any level. The distribution will remain the same.

This alternative approach to the kinematic velocity problem seems to have promise for specialized numerical studies where "global" boundary conditions need to be applied but outflow and inflow might be allowed along some horizontal pressure surface.

7. Conclusion

It has been shown that many alternative objective methods are available for determining kinematic vertical velocities. An analytic solution is found for the second-order adjustment technique. Based on the form of this solution, the simplest realistic correction hypothesis is proposed, i.e., the horizontal divergence estimates are in error by a factor proportional to their distance from the bottom of the atmospheric column. This correction procedure yields excellent results in the practical case of intense mesoscale convection.

The adjustment procedure can be generalized for the case when only global boundary conditions are to be applied or when *any* alternative error proposal is suspected for the atmospheric data.

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