Complex Demodulation

1. The data $X(t)$ is taken to be a nearly-periodic signal plus everything else, $Z(t)$. The amplitude $A$ and phase $\varphi$ of the periodic signal are allowed to be time-dependent but assumed to vary slowly compared to the frequency $\omega$.

$$X(t) = A(t) \cos(\omega t + \varphi(t)) + Z(t)$$

$$= \frac{A(t)}{2} \left[ e^{i(\omega t + \varphi(t))} - e^{-i(\omega t + \varphi(t))} \right] + Z(t)$$

2. "Demodulate" by multiplying by $e^{-i\omega t} \rightarrow Y(t) = X(t)e^{-i\omega t}$, which can be written:

$$Y(t) = \frac{A(t)}{2} e^{i\varphi(t)} + \frac{A(t)}{2} e^{-i(2\omega t + \varphi(t))} + Z(t)e^{-i\omega t}$$

(a) (b) (c)

Term (a) varies slowly, with no power at or above frequency $\omega$;
Term (b) varies at frequency $2\omega$;
Term (c) varies at frequency $\omega$. (Note that by postulate $Z(t)$ has no power at frequency $\omega$, so the shifted term (c) has no power at zero frequency.)

3. Low-pass filter to remove frequencies at or above frequency $\omega$.
   This (nearly) removes terms (b) and (c), and smooths (a). The result is:

$$Y'(t) \approx \frac{A'(t)}{2} e^{i\varphi'(t)}, \text{ where the prime indicates smoothing.}$$

The choice of smoother determines the width of the frequency band retained. For triangle smoothing with length $(2T - 1)$, where $T$ is the demodulation period, $T = 2\pi/\omega$, the half-power bandwidth is from $T/(1+0.44295)$ to $T/(1-0.44295)$, or from about $0.69T$ to $1.8T$.

4. Extract $A'$ and $\varphi'$: $A'(t) = 2|Y'| = 2 \left( \text{Re}\{Y'\}^2 + \text{Im}\{Y'\}^2 \right)^{1/2}$, $\varphi'(t) = \text{atan} \left( \frac{\text{Im}\{Y'\}}{\text{Re}\{Y'\}} \right)$

5. The choice of $\omega$ can be checked by locally fitting a line to the phase: $\varphi(t) \approx \gamma t + b$.
   Typically this is done over running intervals of length $T$. With the origin chosen at the central time of each interval (so $b=0$), this gives $\cos(\omega t + \varphi) = \cos(\omega t + \gamma t) = \cos(\omega^* t)$. The fitted frequency $\omega^*(t) = \omega + \gamma$ is a check on the initial choice $\omega$.